Spin 1/2 reference sheet

$$S_z|+\rangle = +\frac{\hbar}{2}|+\rangle \qquad\qquad S_x|+\rangle_x = +\frac{\hbar}{2}|+\rangle_x \qquad\qquad S_y|+\rangle_y = +\frac{\hbar}{2}|+\rangle_y$$

$$S_z|-\rangle = -\frac{\hbar}{2}|-\rangle \qquad\qquad S_x|-\rangle_x = -\frac{\hbar}{2}|-\rangle_x \qquad\qquad S_y|-\rangle_y = -\frac{\hbar}{2}|-\rangle_y$$

$$\langle+|+\rangle = \langle-|-\rangle = 1 \qquad |+\rangle_x = \frac{1}{\sqrt{2}}\big[|+\rangle + |-\rangle\big] \qquad |+\rangle_y = \frac{1}{\sqrt{2}}\big[|+\rangle + i|-\rangle\big]$$

$$\langle+|-\rangle = \langle-|+\rangle^* = 0 \qquad |-\rangle_x = \frac{1}{\sqrt{2}}\big[|+\rangle - |-\rangle\big] \qquad |-\rangle_y = \frac{1}{\sqrt{2}}\big[|+\rangle - i|-\rangle\big]$$

$$\hat{\mathbf{n}} = \hat{\mathbf{i}}\sin\theta\cos\phi + \hat{\mathbf{j}}\sin\theta\sin\phi + \hat{\mathbf{k}}\cos\theta$$

$$S_n = S_x \sin\theta\cos\phi + S_y \sin\theta\sin\phi + S_z \cos\theta$$

$$|+\rangle_n = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\rangle$$

$$|-\rangle_n = \sin\frac{\theta}{2}|+\rangle - \cos\frac{\theta}{2}e^{i\phi}|-\rangle$$

Using the S_z representation: ($\doteq$ means "is represented by")

$$S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_n \doteq \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}$$

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Pauli Matrices:

$$\vec{\mathbf{S}} \doteq \frac{\hbar}{2}\vec{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad\qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad\qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x|\pm\rangle_x = \pm|\pm\rangle_x \qquad\qquad \sigma_y|\pm\rangle_y = \pm|\pm\rangle_y \qquad\qquad \sigma_z|\pm\rangle = \pm|\pm\rangle$$

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Spin 1 reference sheet

$$S_z|1\rangle = \hbar|1\rangle \qquad S_z|0\rangle = 0 \qquad S_z|-1\rangle = -\hbar|-1\rangle$$

$$|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle \qquad\qquad |1\rangle_y = \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle \qquad\qquad |0\rangle_y = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle$$

$$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle \qquad\qquad |-1\rangle_y = \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|1\rangle_n = \frac{1+\cos\theta}{2}e^{-i\phi}|1\rangle + \frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1-\cos\theta}{2}e^{i\phi}|-1\rangle$$

$$|0\rangle_n = -\frac{\sin\theta}{\sqrt{2}}e^{-i\phi}|1\rangle + \cos\theta|0\rangle + \frac{\sin\theta}{\sqrt{2}}e^{i\phi}|-1\rangle$$

$$|-1\rangle_n = \frac{1-\cos\theta}{2}e^{-i\phi}|1\rangle - \frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1+\cos\theta}{2}e^{i\phi}|-1\rangle$$

Using the S_z representation: ($\overset{\bullet}{=}$ means "is represented by")

$$S_z \overset{\bullet}{=} \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad S_x \overset{\bullet}{=} \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y \overset{\bullet}{=} \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$S_n \overset{\bullet}{=} \hbar \begin{pmatrix} \cos\theta & \frac{\sin\theta}{\sqrt{2}}e^{-i\phi} & 0 \\ \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & 0 & \frac{\sin\theta}{\sqrt{2}}e^{-i\phi} \\ 0 & \frac{\sin\theta}{\sqrt{2}}e^{i\phi} & -\cos\theta \end{pmatrix}$$

$$|1\rangle \overset{\bullet}{=} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad\qquad |0\rangle \overset{\bullet}{=} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad\qquad |-1\rangle \overset{\bullet}{=} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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