Using Great Circles to Understand Motion on a Rotating Sphere

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NSF - Paradigms
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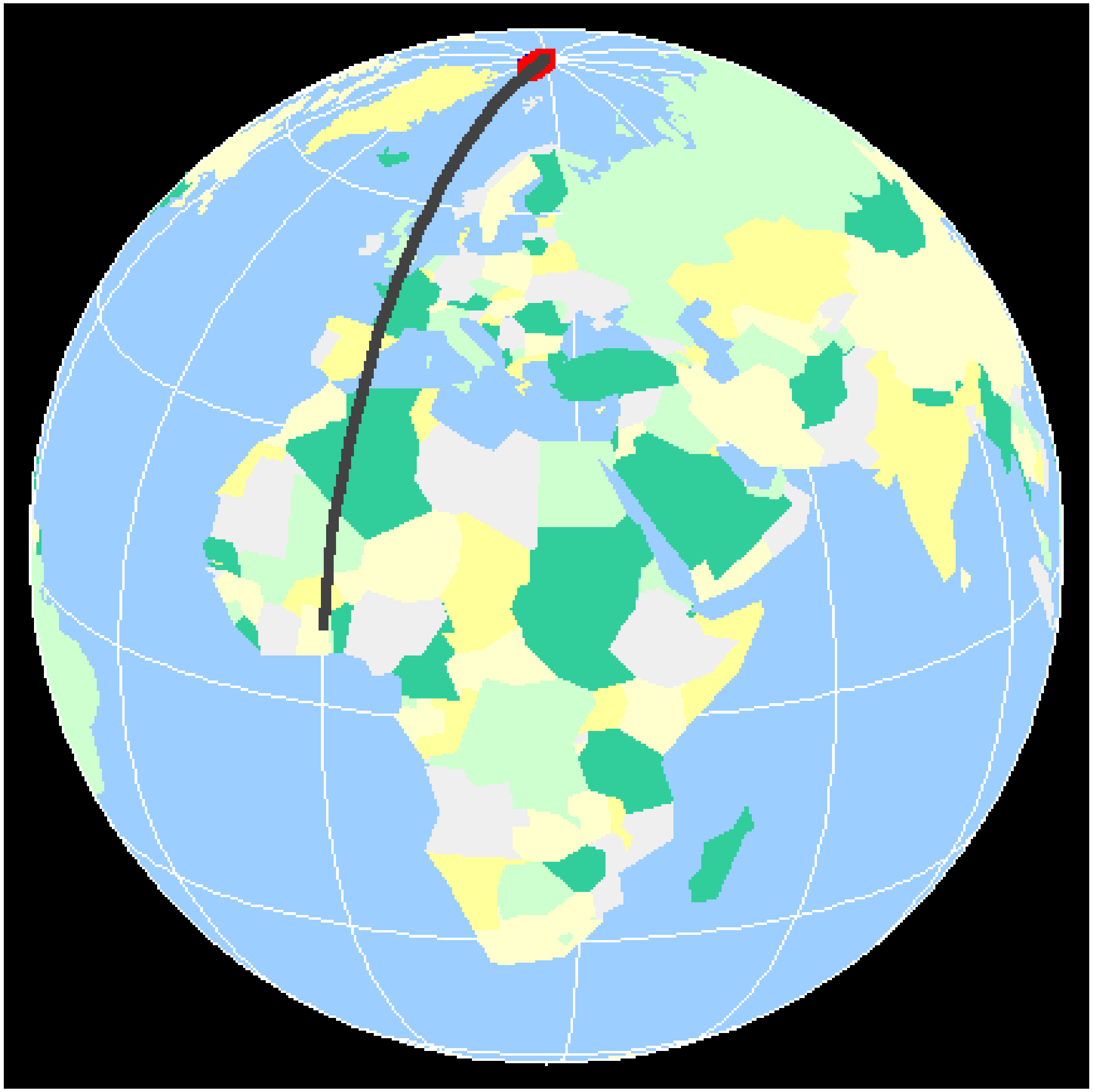
Rotating reference frames

$$\left(\frac{d\vec{\mathbf{r}}}{dt}\right)_{\text{inertial}} = \left(\frac{d\vec{\mathbf{r}}}{dt}\right)_{\text{rotating}} + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$$

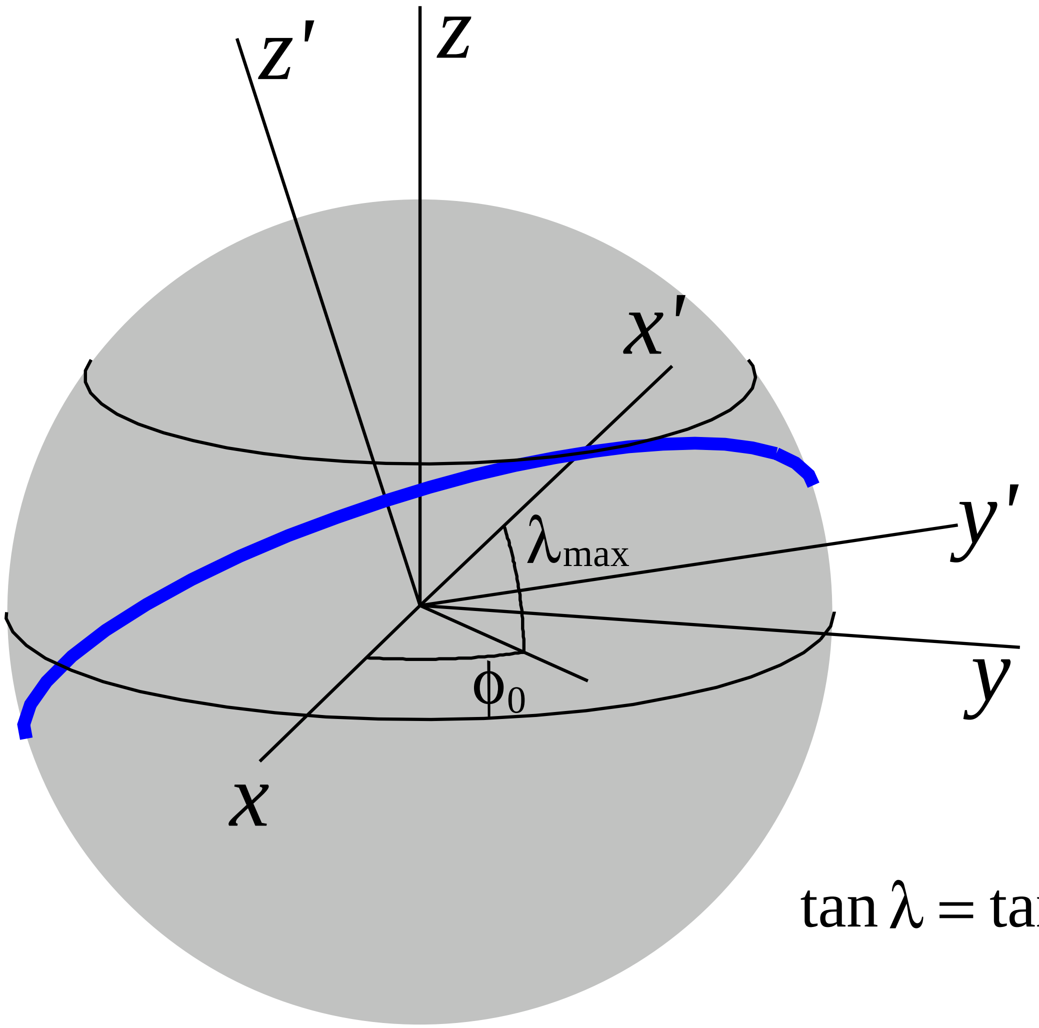
$$m\left(\frac{d^2\vec{\mathbf{r}}}{dt^2}\right)_{\text{rotating}} = \vec{\mathbf{F}} - m\vec{\boldsymbol{\omega}} \times (\vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}) - 2m\vec{\boldsymbol{\omega}} \times \left(\frac{d\vec{\mathbf{r}}}{dt}\right)_{\text{rotating}}$$

$$\vec{\mathbf{a}}_{\text{cent}} = -\vec{\boldsymbol{\omega}} \times (\vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}) \qquad \vec{\mathbf{a}}_{\text{cor}} = -2\vec{\boldsymbol{\omega}} \times \vec{\mathbf{v}}_r$$

Puck launched
from North Pole



Great circle coordinate systems



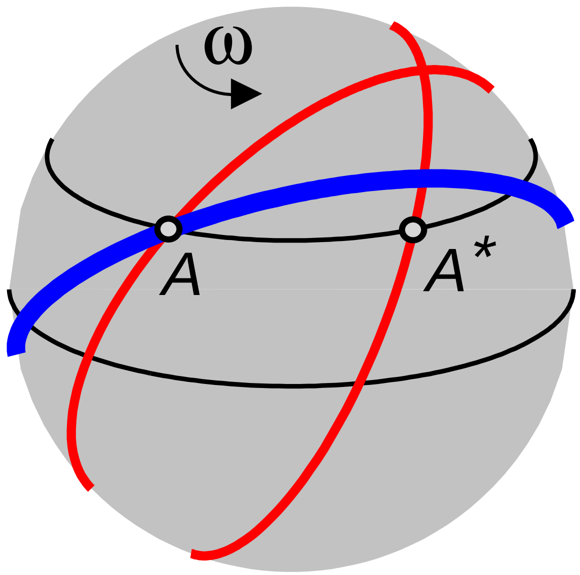
$$0 \le \phi_0 \le 2\pi$$

$$0 \le \lambda_{max} \le \pi/2$$

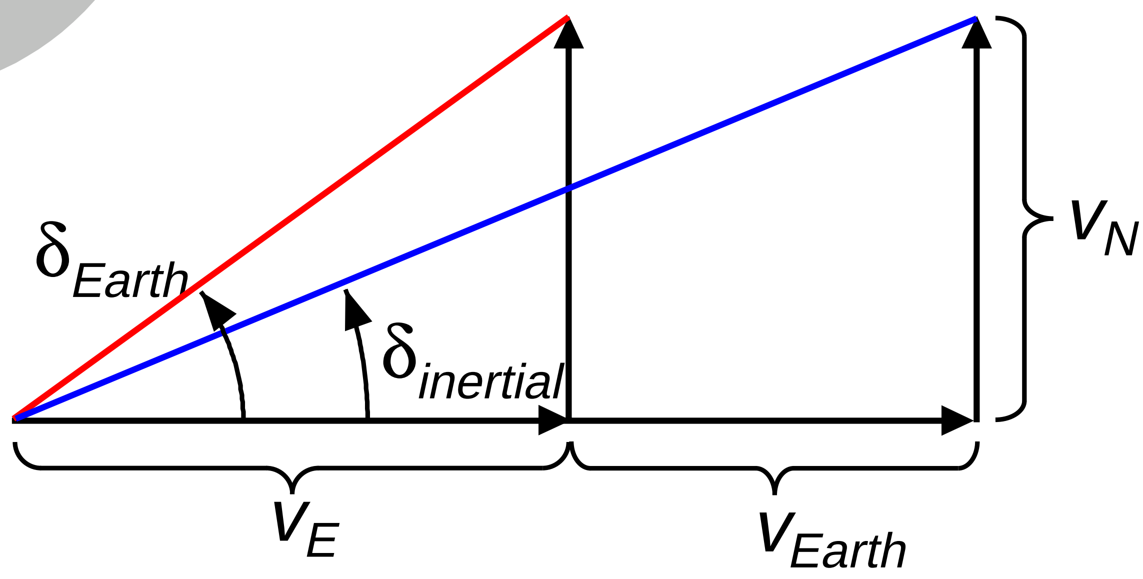
$$\lambda' = 0$$

$$\tan\lambda = \tan\lambda_{max}\cos(\phi - \phi_0)$$

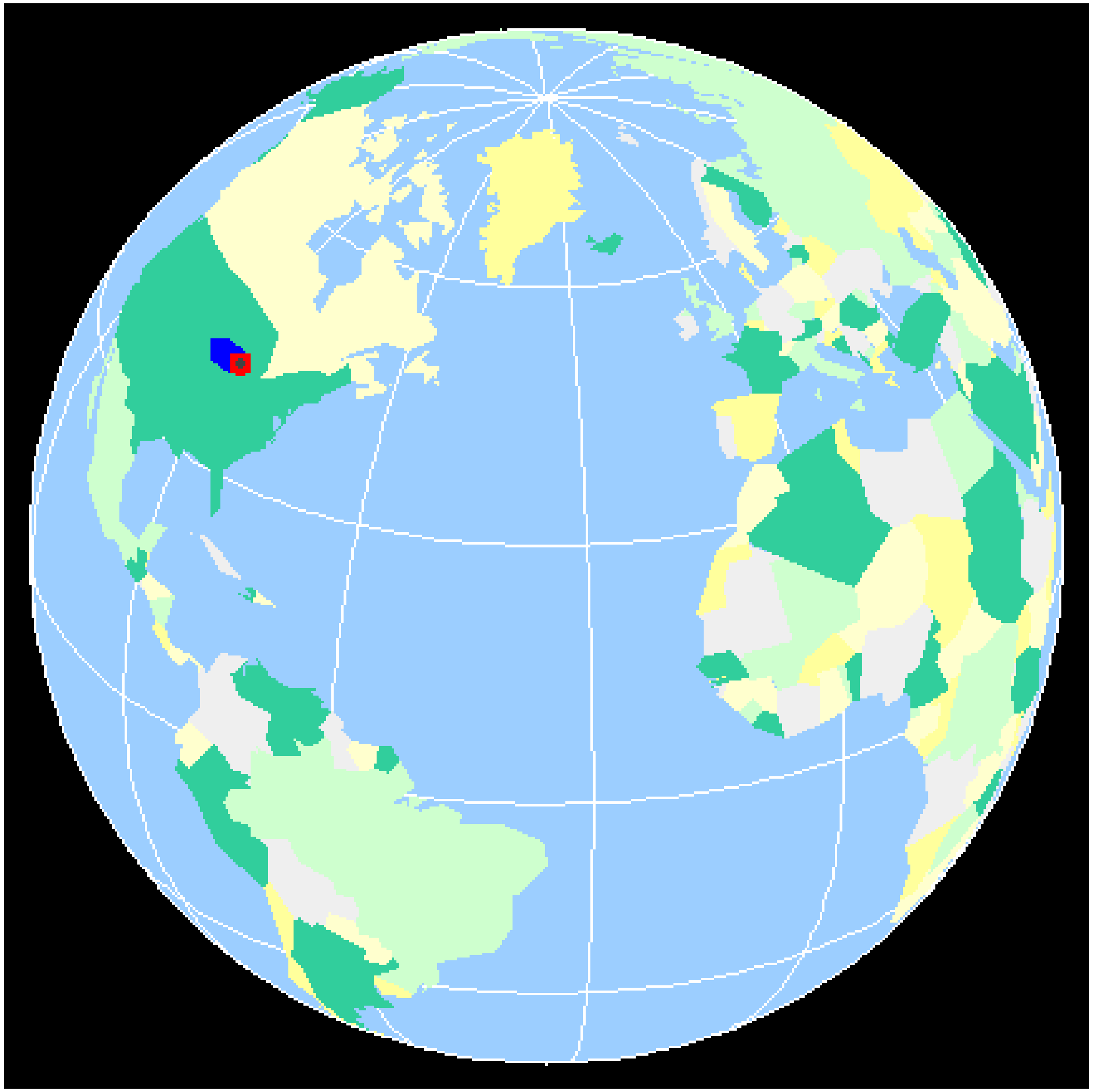
Inertial and earthbound great circles



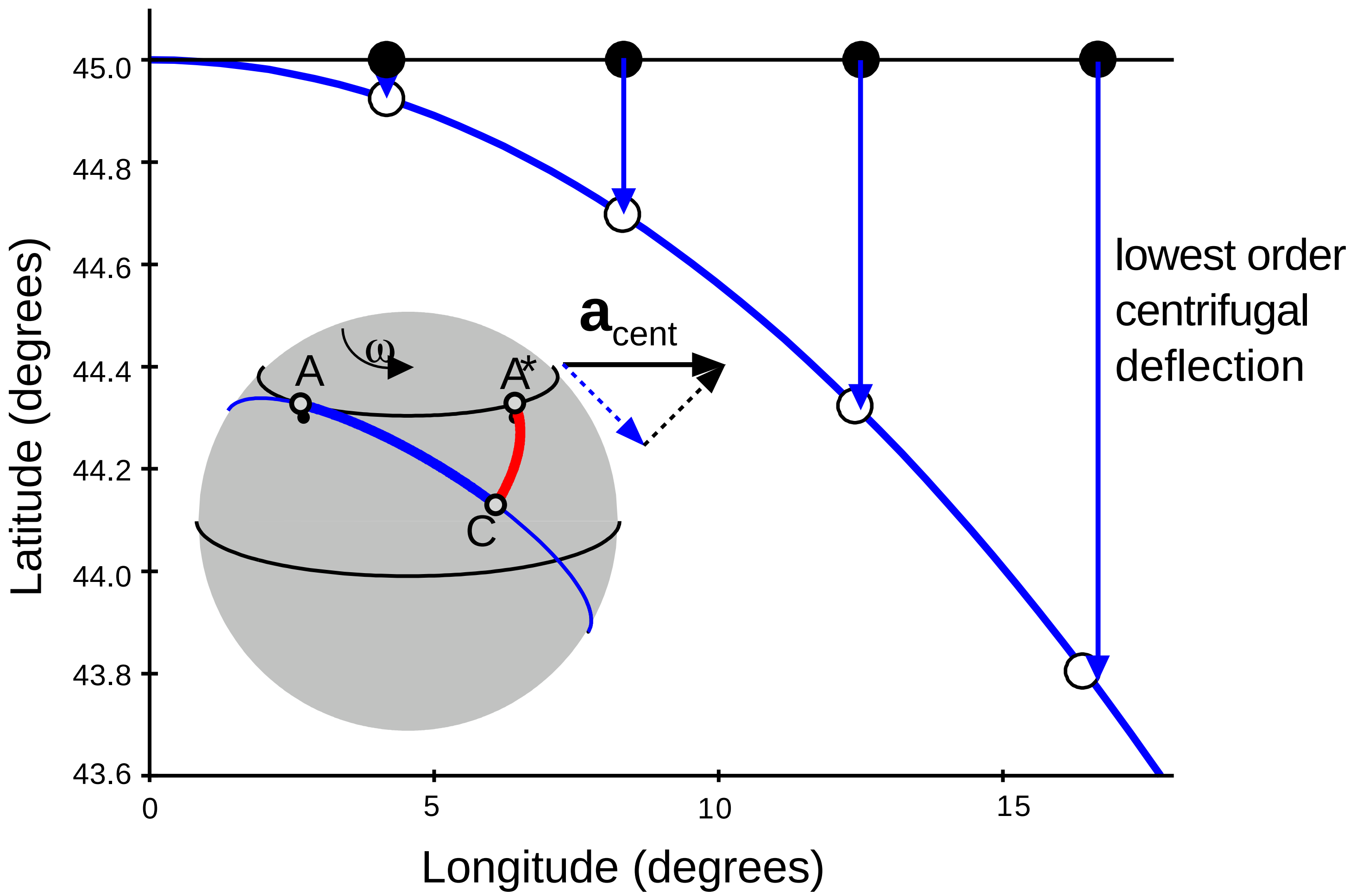
$$\Omega = \frac{v}{R} = \frac{\sqrt{\left(v_E + \omega R \cos\lambda_{\text{start}}\right)^2 + v_N^2}}{R}$$



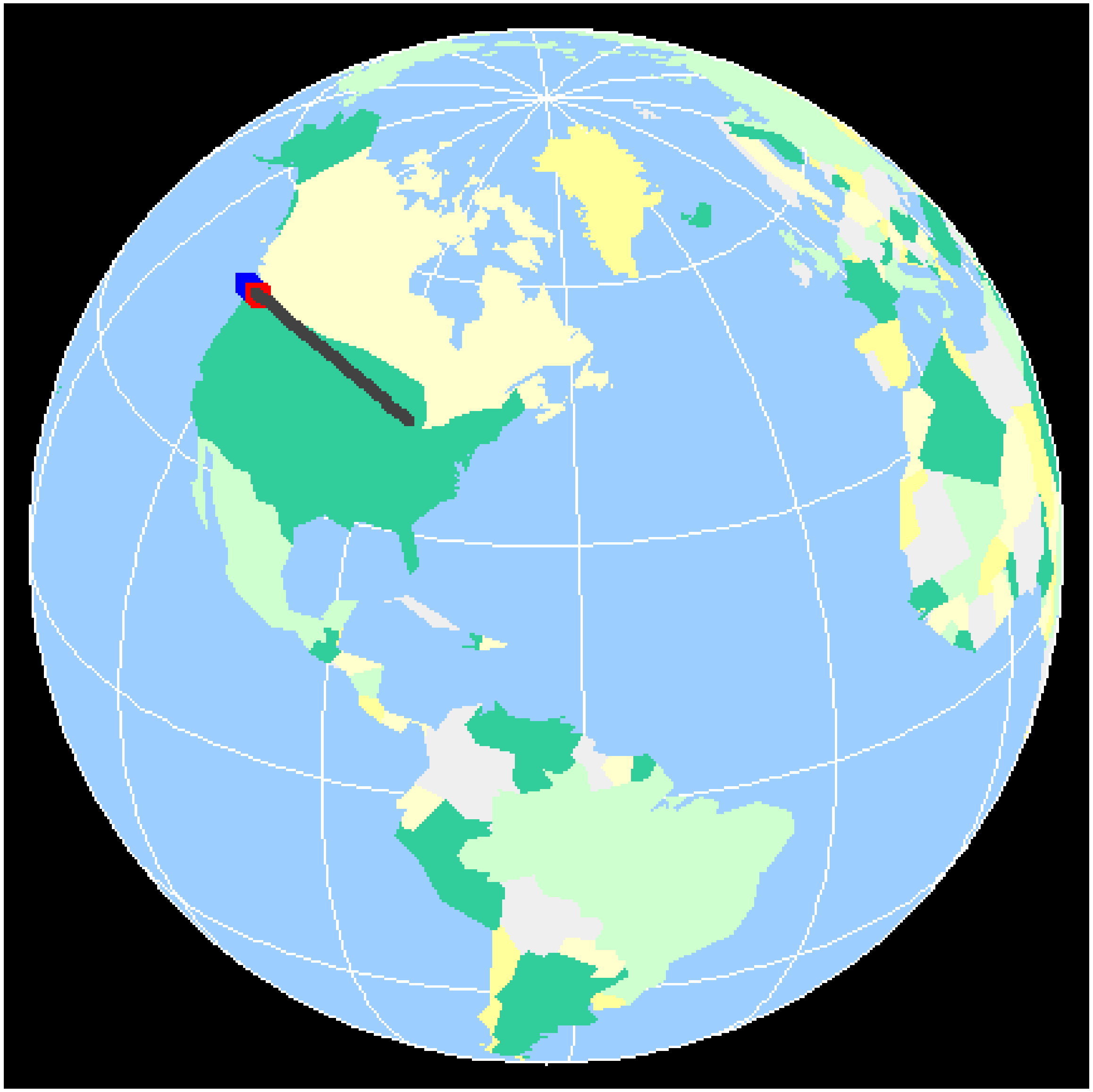
Puck released
from rest



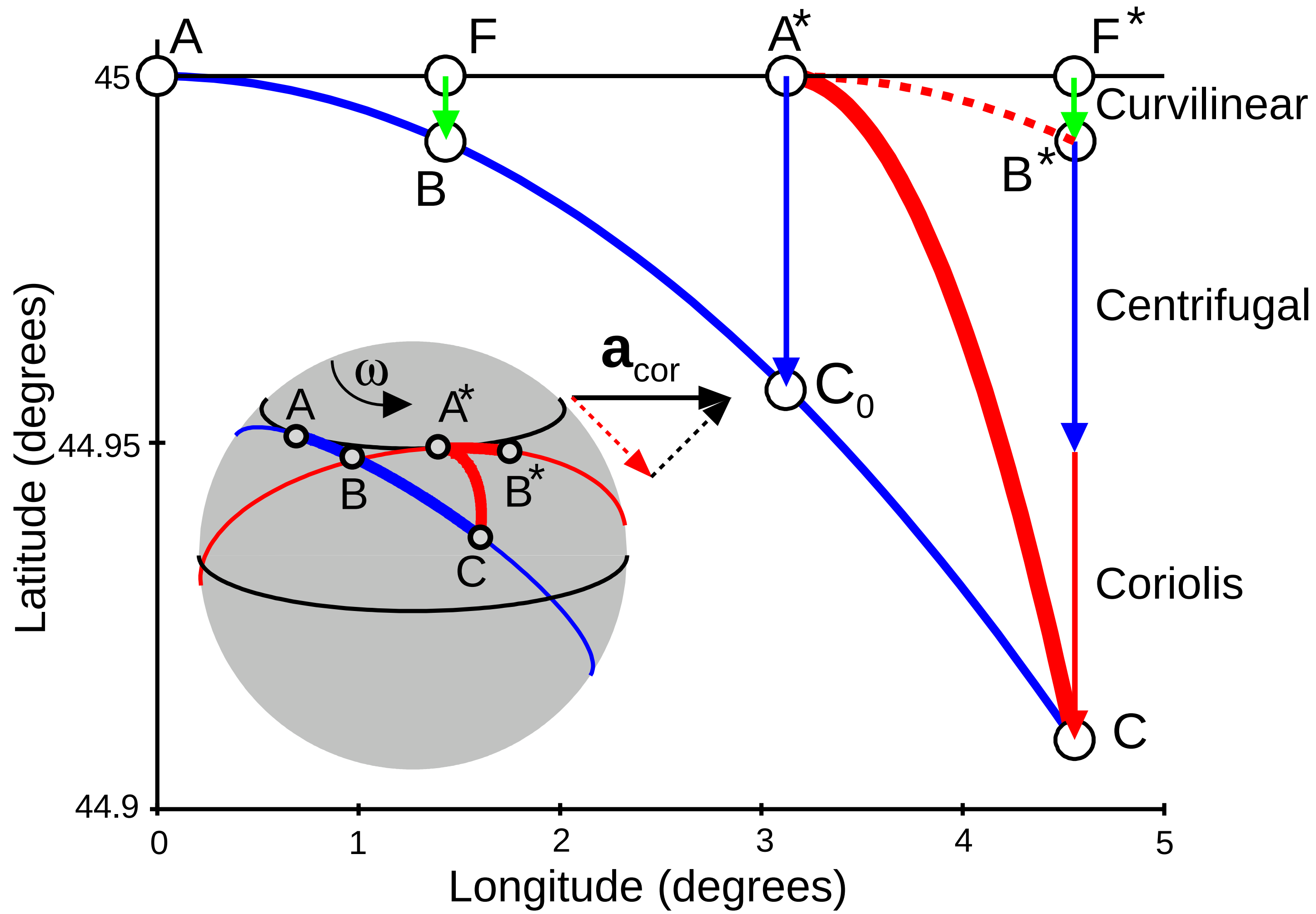
Puck initially at rest



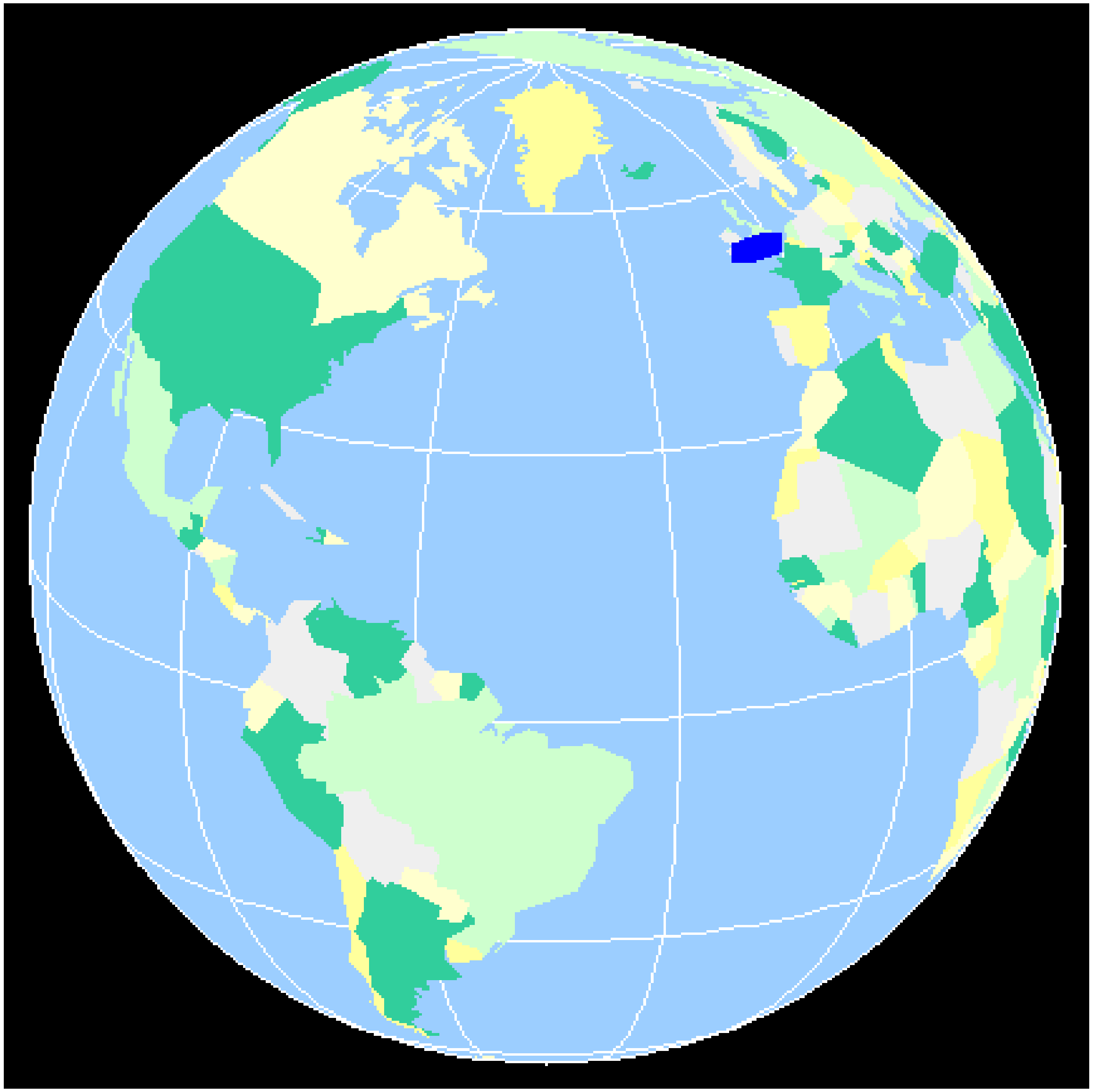
Puck launched toward east



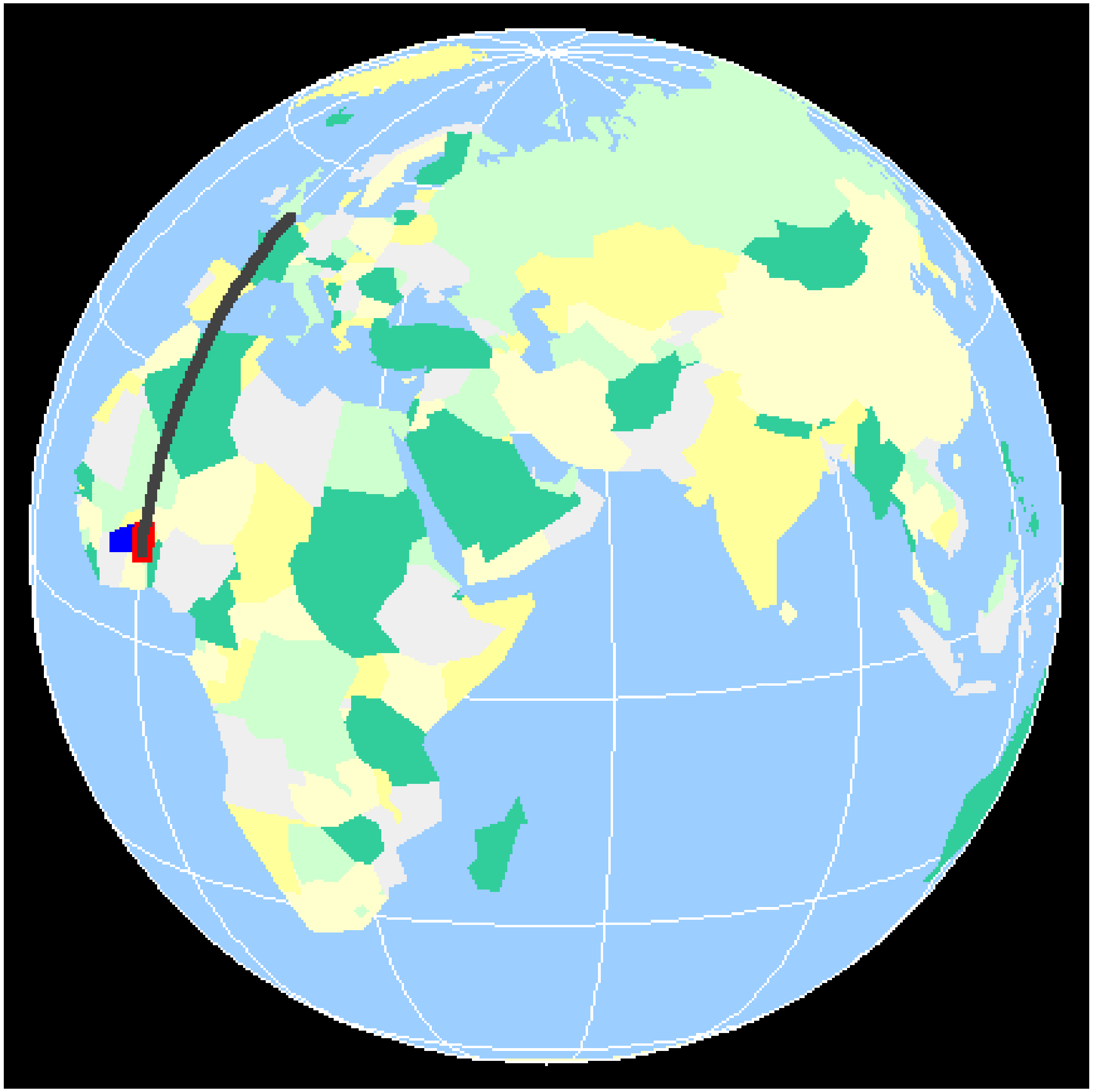
Puck launched eastward



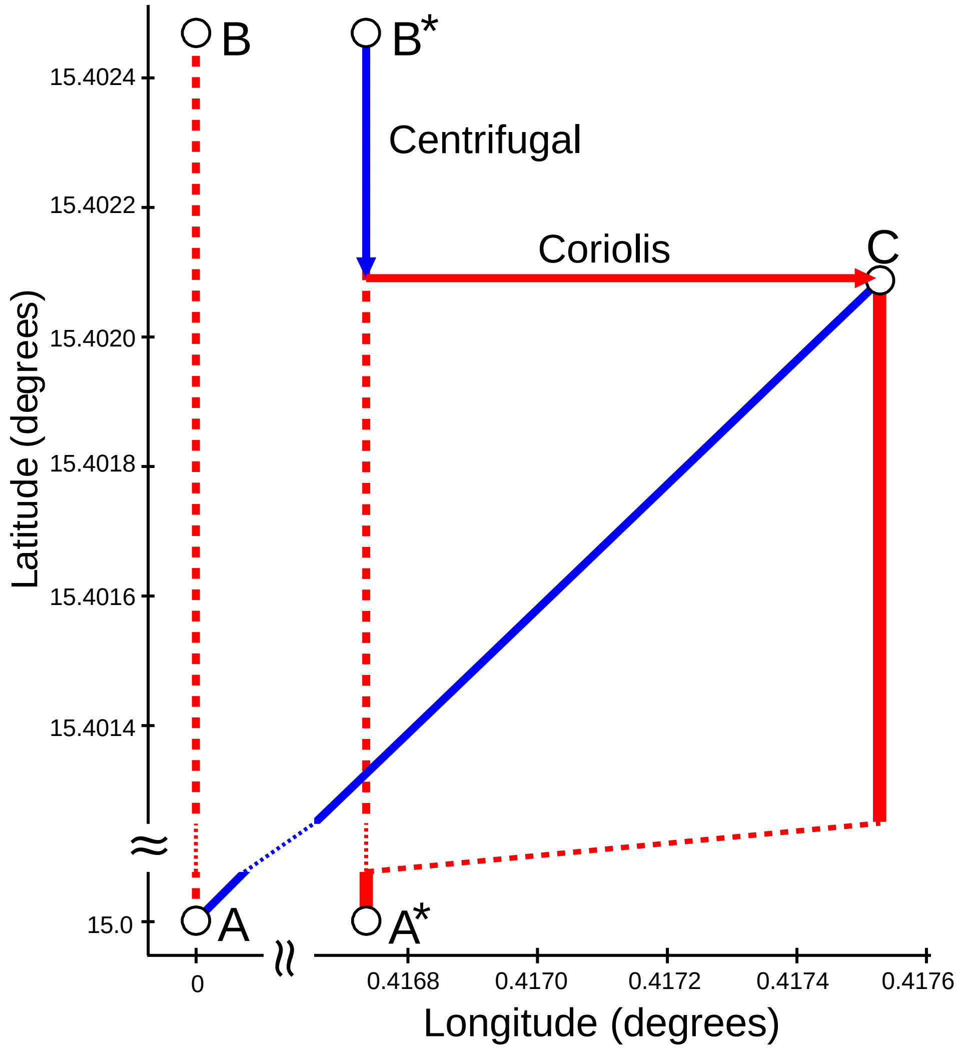
Puck launched toward west from London (earth at rest)

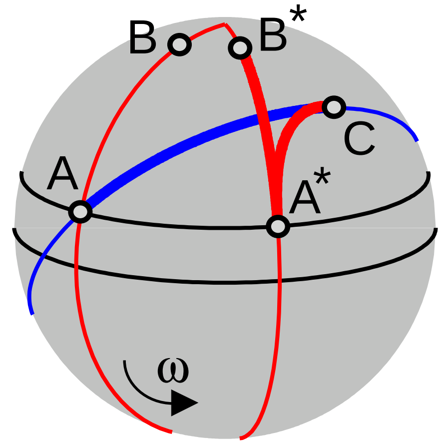


Puck launched toward north



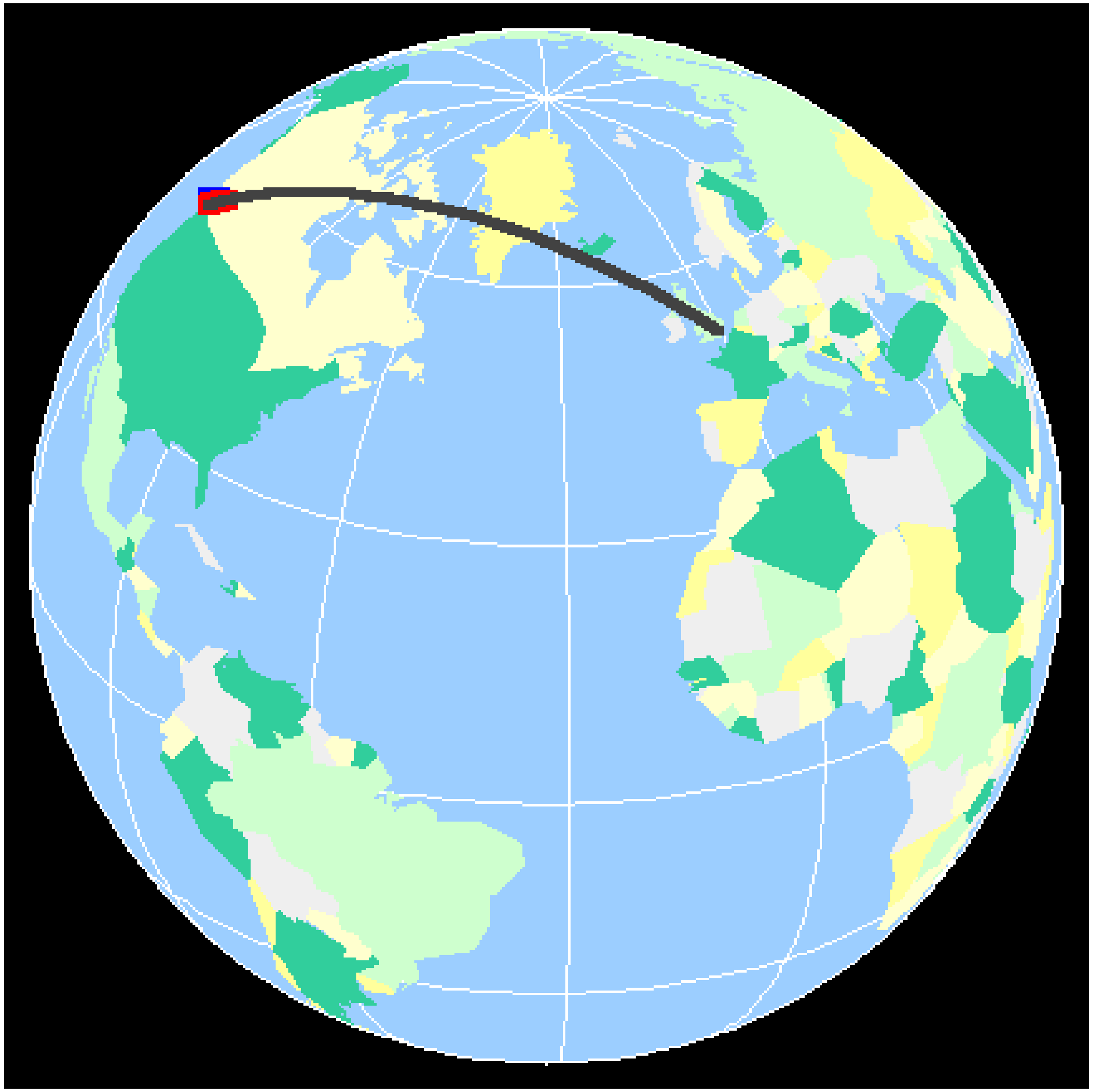
Puck launched northward





- B moves away
- v_{Earth} ↓
- L conserved

Puck launched
from Vancouver
to London



$v = 1350$ mi/hr

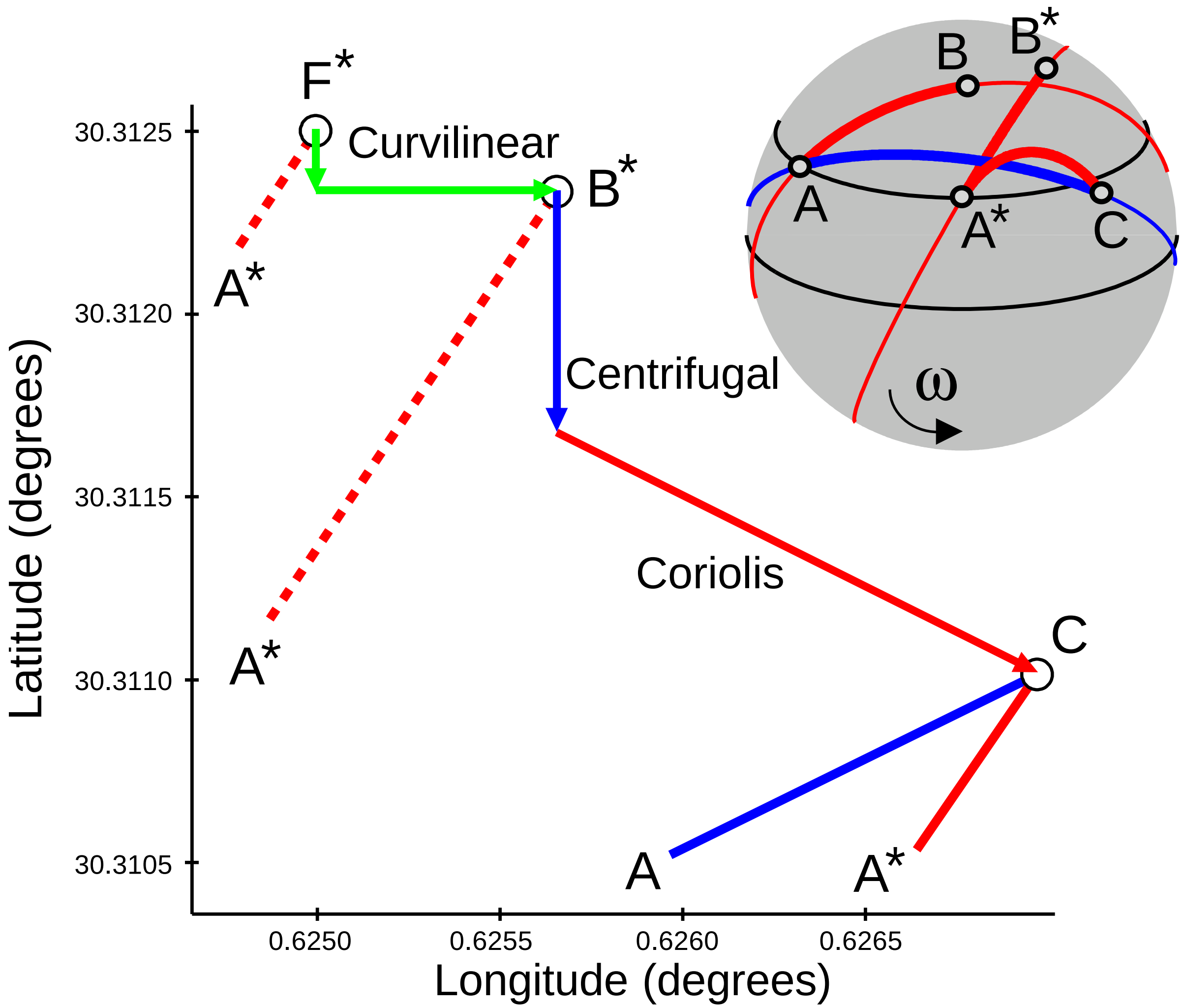
$t = 3.5$ hr

$a \approx \omega v \approx 0.5\%\ g$

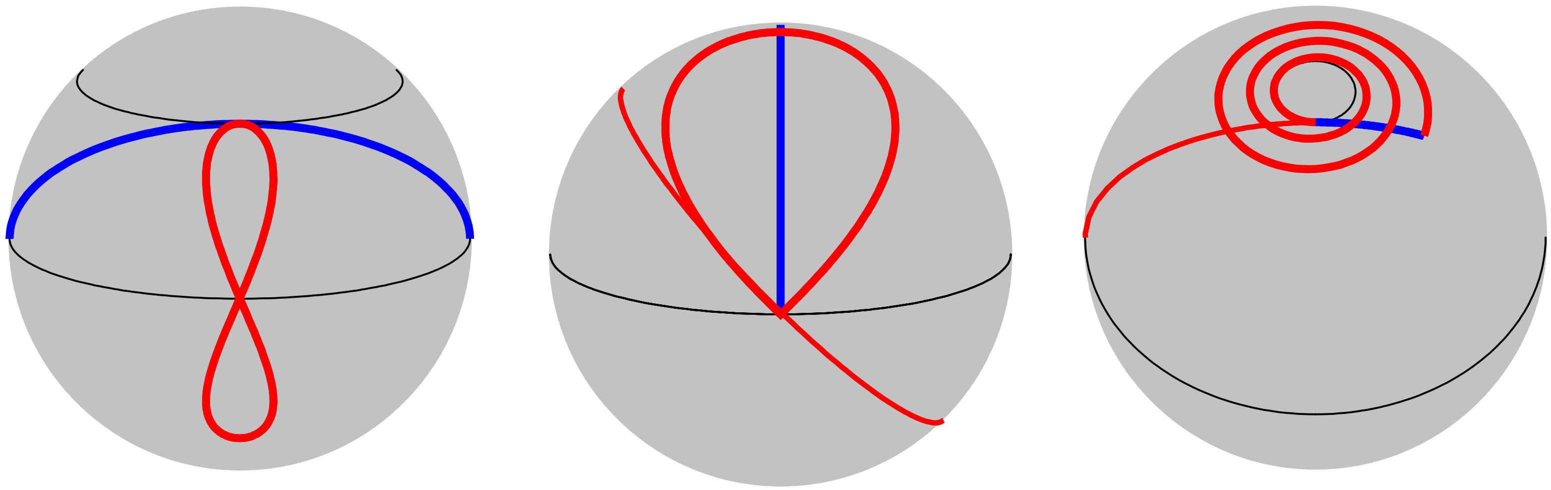
$a \approx 350$ mi/hr/hr

$D \approx at^2/2 \approx 2000$ mi

Puck launched generally



Interesting earthbound paths

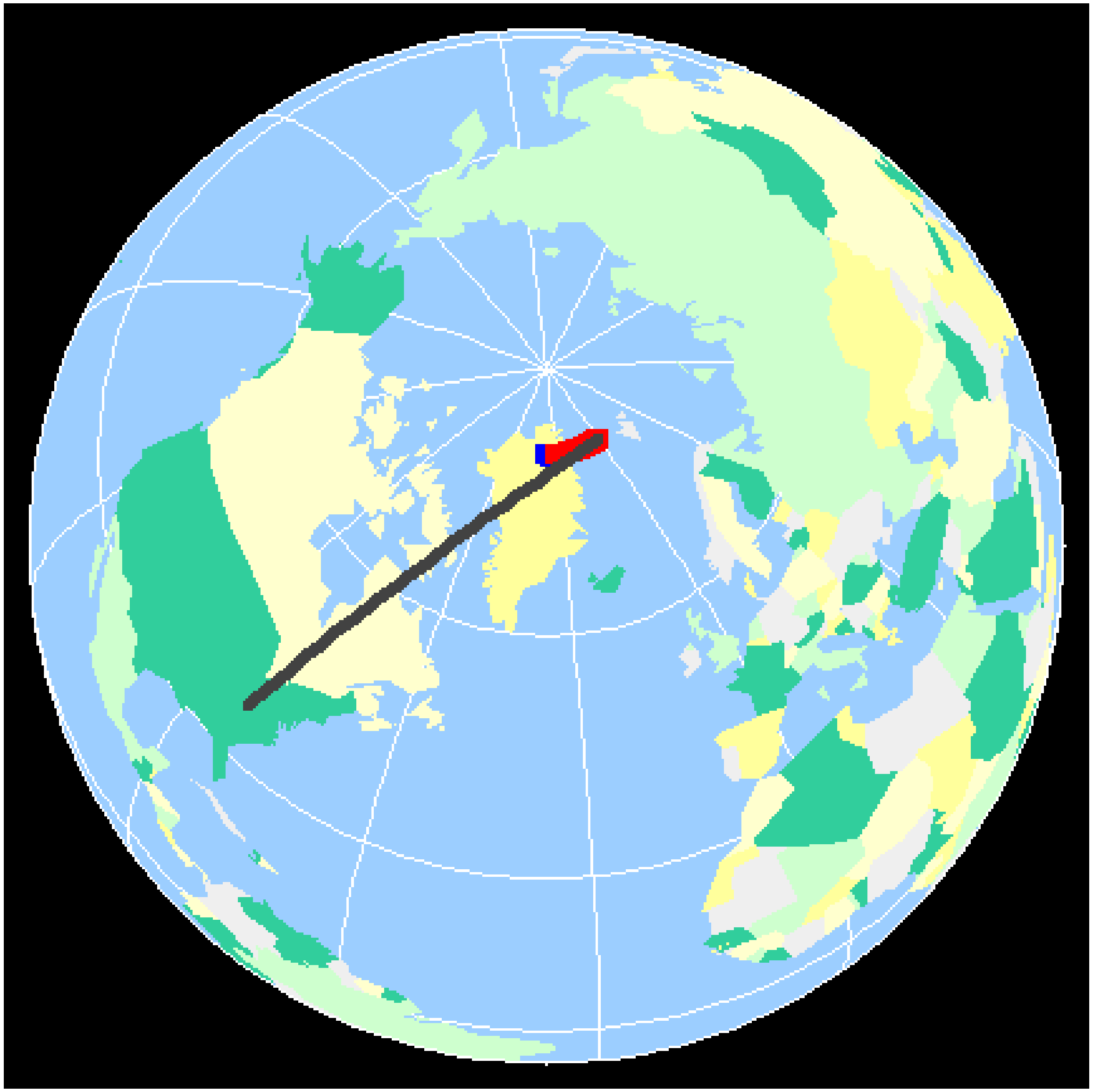


- Launch East
- $v_{inertial} = v_{Equator}$

- Launch NW
- $v_{inertial} = v_{Equator}$

- Launch West
- $v_{inertial} \approx v_{Earth}/6$

Puck launched to West



Summary

- Great circles aid understanding of inertial forces on sphere
- Animations on web at www.physics.orst.edu/~mcintyre/coriolis
- Am. J. Phys. **68**, 1097 (2000). (Dec. 2000)