1. a) $c \cong 3 \times 10^8 \ m/s$

b) $\lambda_{visible} \simeq 400 \text{ nm} \rightarrow 700 \text{ nm}$

c) $\vec{E} = E_0 \left(\frac{\hat{x}}{3} + \frac{2\hat{y}}{3} + \frac{2\hat{z}}{3} \right) \exp\left\{ i \left[k \left(\frac{y}{\sqrt{2}} - \frac{z}{\sqrt{2}} \right) - \omega t \right] \right\}$

$\vec{k} \times \vec{E} = \omega \vec{B}$ $\qquad\qquad \vec{k} = k \left(\frac{\hat{y}}{\sqrt{2}} - \frac{\hat{z}}{\sqrt{2}} \right)$

$\Rightarrow \vec{B}_0 = \frac{1}{\omega} \vec{k} \times \vec{E}_0$

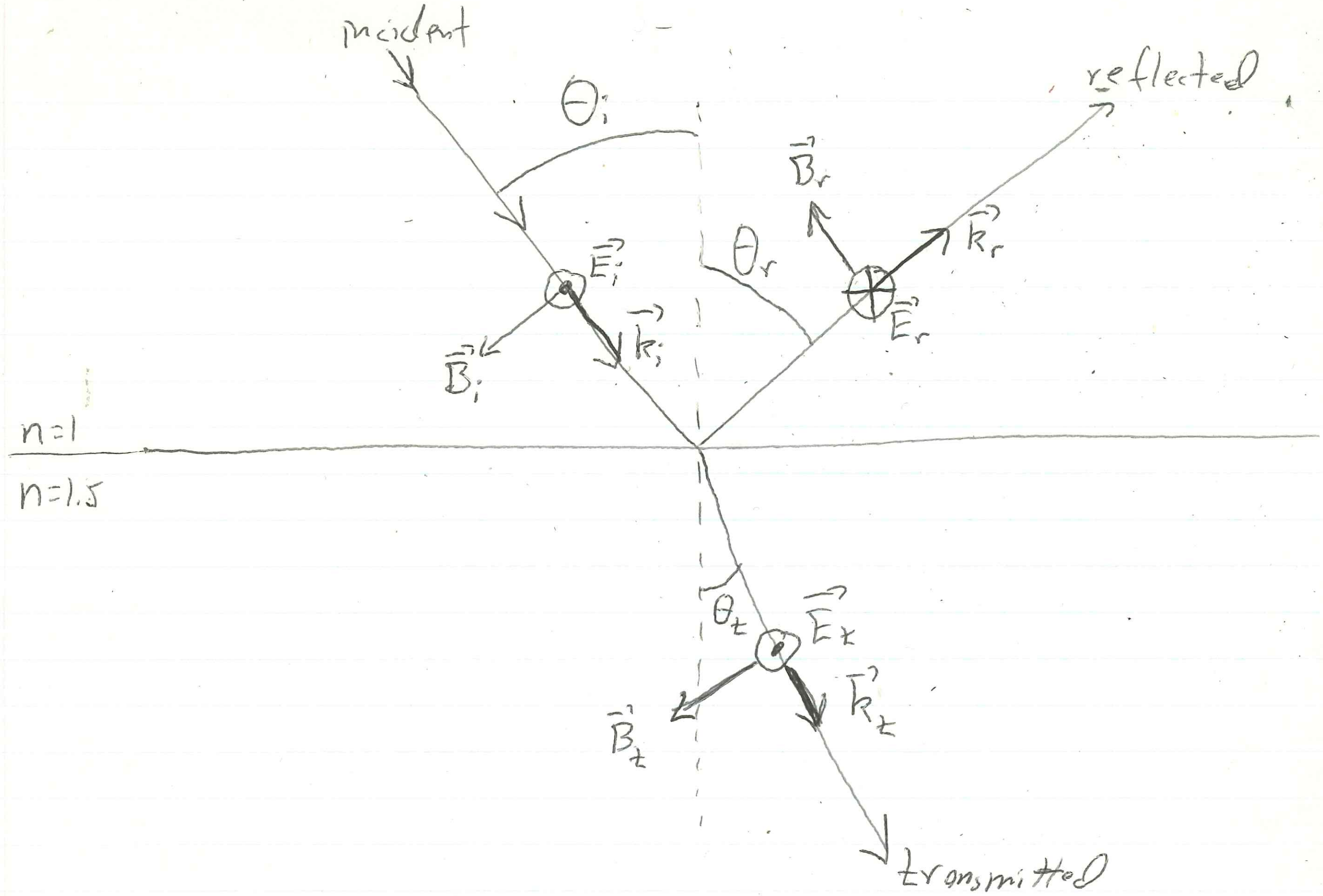
$$= \frac{1}{\omega} k E_0 \cdot \frac{1}{3} \cdot \frac{1}{\sqrt{2}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & -1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$\vec{B}_0 = \frac{k E_0}{\omega 3\sqrt{2}} \left[\hat{x}(4) + \hat{y}(-1) + \hat{z}(-1) \right]$$ $\qquad \omega = 2\pi f = 2\pi \frac{c}{\lambda} = kc$

$$= \frac{E_0}{c} \frac{1}{3\sqrt{2}} (4\hat{x} - \hat{y} - \hat{z})$$

$$\Rightarrow \boxed{\vec{B} = \frac{E_0}{c} \frac{1}{3\sqrt{2}} (4\hat{x} - \hat{y} - \hat{z}) \exp\left\{ i \left[k \left(\frac{y}{\sqrt{2}} - \frac{z}{\sqrt{2}} \right) - \omega t \right] \right\}}$$

d)



Note that $\vec{E}_r$ has π phase shift w.r.t. $\vec{E}_i$ since light is going from less to more dense medium.

2. $f = 6 \times 10^{14}$ Hz

a) $36°$ is $\frac{1}{10}$ of $360°$ or 2π

$\Rightarrow$ distance between $36°$ phase points $= d = \frac{\lambda}{10}$

$c = \lambda f \Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{14} \text{ s}^{-1}} = \frac{1}{2} \times 10^{-6}$

$\lambda = \frac{1}{2} \mu m = 500$ nm

$\Rightarrow d = \frac{\lambda}{10} = \frac{500 \text{ nm}}{10} = 50$ nm

b) Phase of Light field is $\phi = kx - \omega t$

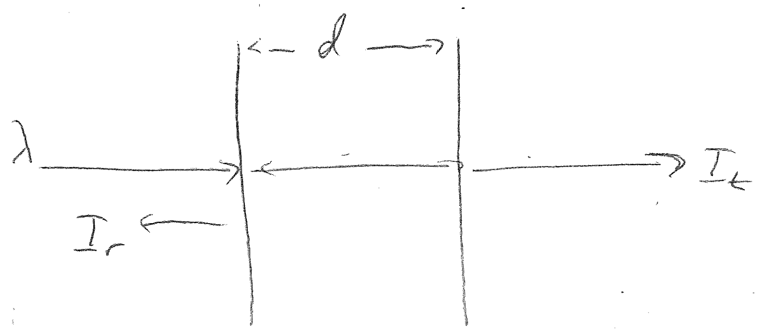
for fixed x : $\Delta\phi = \omega \Delta t$ (ignore sign)

$\Rightarrow \Delta\phi = 2\pi f \Delta t = 2\pi \times 6 \times 10^{14} \text{ s}^{-1} \times 1 \times 10^{-6} \text{ s}$

$= 2\pi \times 6 \times 10^8$ rad

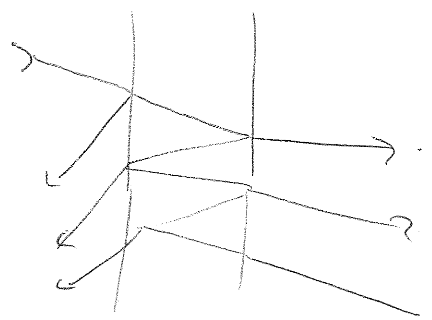
# of waves $= \frac{\Delta\phi}{2\pi} = 6 \times 10^8$

3



The reflected and transmitted beams are each composed of the multiple beams reflected or transmitted within the etalon.

i.e.



a) For normal incidence the path length difference between successive waves is

$$\Lambda = 2d$$

The maximum in transmission occurs when each successive wave has a $2\pi m$ phase shift:

$$\Rightarrow \quad \delta = k\Lambda = 2\pi m \qquad m = \text{integer}$$

Reflection is a minimum since first reflection is π out of phase with all others since $r = -r'$

$\Rightarrow$ resonance condition is

$$\frac{2\pi}{\lambda} 2d = 2\pi m$$

$$\Rightarrow \boxed{d = m \cdot \frac{\lambda}{2}}$$

b) $d = m_1 \frac{\lambda_1}{2} = m_2 \frac{\lambda_2}{2}$

assume N fringes of λ_1 pass by before reaching d'

$\Rightarrow d' = (m_1 + N)\frac{\lambda_1}{2}$

if $\lambda_2 > \lambda_1$ then $N-1$ fringes of λ_2 will pass by

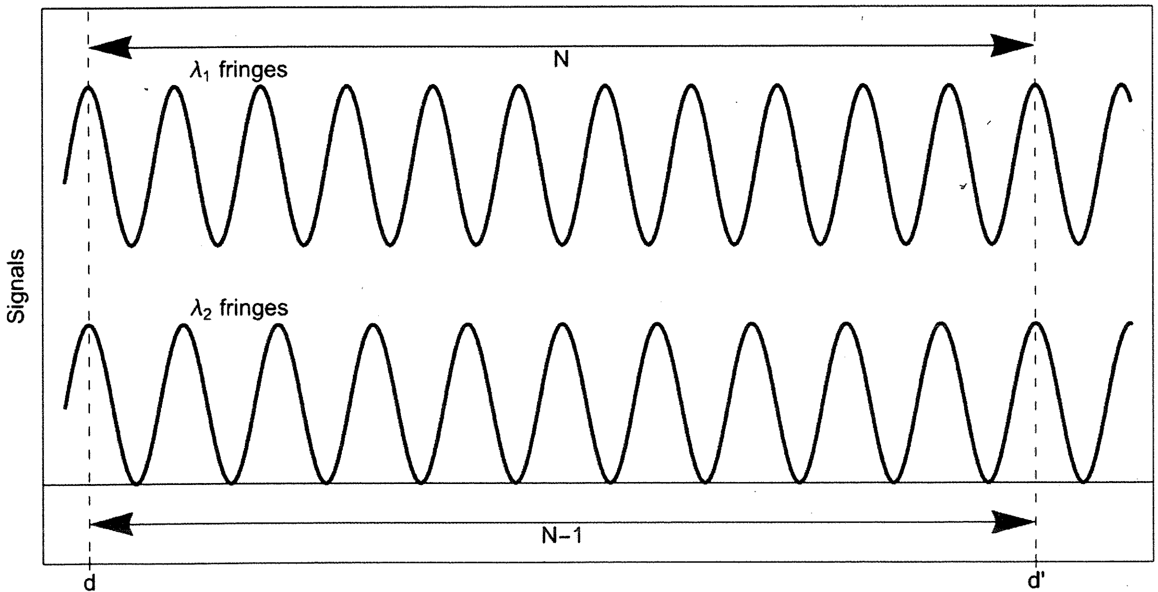
See graph below

$\Rightarrow d' = (m_1 + N)\frac{\lambda_1}{2} = (m_2 + N - 1)\frac{\lambda_2}{2}$

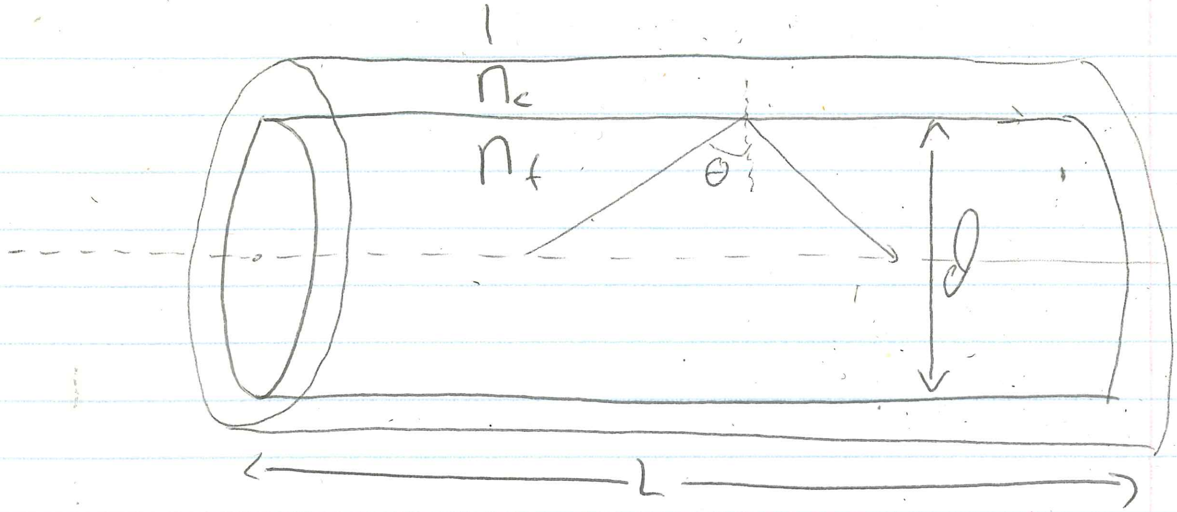
$d' - d = N\frac{\lambda_1}{2} = (N-1)\frac{\lambda_2}{2}$

$\Rightarrow N\left(\frac{\lambda_2}{2} - \frac{\lambda_1}{2}\right) = \frac{\lambda_2}{2}$

$$\boxed{N = \frac{\lambda_2}{\lambda_2 - \lambda_1}}$$

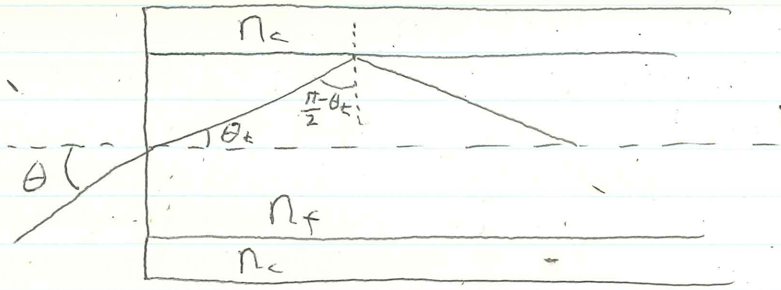


4



a) Need total internal reflection.

$\Rightarrow$ need $n_f > n_c$ so reflection at fiber-cladding interface is internal b)



To have wave TIR'ed, need $\frac{\pi}{2} - \theta_t \geq \theta_c$

Find θ_c: condition $n_f \sin\theta_c = n_c \sin\frac{\pi}{2} = n_c$

$$\Rightarrow \sin\theta_c = \frac{n_c}{n_f}$$

At fiber entrance $\sin\theta = n_f \sin\theta_t$

θ_{max} is when $\frac{\pi}{2} - \theta_t = \theta_c$

$$\Rightarrow \sin\theta_{max} = n_f \sin\left(\frac{\pi}{2} - \theta_c\right) = n_f \cos\theta_c$$

$$\sin\theta_{max} = n_f\sqrt{1-\sin^2\theta_c} = n_f\sqrt{1-\frac{n_c^2}{n_f^2}}$$

$$\sin\theta_{max} = \sqrt{n_f^2 - n_c^2}$$

$$\boxed{\theta_{max} = \sin^{-1}\left(\sqrt{n_f^2-n_c^2}\right)}$$