4.2.1 The operators are (I have included $\hbar$, but OK if not there)

$$L_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad L_y \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad L_z \doteq \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(1) Possible values of L_z must be eigenvalues. L_z is already diagonal, so eigenvalues can be read off by inspection:

$$L_z = \hbar, 0, -\hbar \quad \text{(or } 1, 0, \text{-1)}$$

(2) Initial state is $|\psi\rangle = |L_z = \hbar\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. Find expectation values:

$$\langle L_x \rangle = \langle \psi | L_x | \psi \rangle = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$\langle L_x^2 \rangle = \langle \psi | L_x^2 | \psi \rangle = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \frac{\hbar^2}{2}$$

The uncertainty is

$$\Delta L_x = \sqrt{\langle L_x^2 \rangle - \langle L_x \rangle^2} = \sqrt{\frac{\hbar^2}{2} - 0} = \frac{\hbar}{\sqrt{2}}$$

(3) For L_x the diagonalization yields the eigenvalues

$$L_x \doteq \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad -\lambda\left(\lambda^2 - \tfrac{\hbar^2}{2}\right) - \tfrac{\hbar}{\sqrt{2}}\left(-\lambda \tfrac{\hbar}{\sqrt{2}}\right) = 0$$

$$\lambda\left(\lambda^2 - \hbar^2\right) = 0 \quad \Rightarrow \quad \lambda = 1\hbar, 0, -1\hbar$$

and the eigenvectors

$$\frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 1\hbar\begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{matrix} b = a\sqrt{2} \\ a + c = b\sqrt{2} \\ b = c\sqrt{2} \end{matrix}$$

$$|a|^2 + |b|^2 + |c|^2 = 1 \Rightarrow |b|^2\left(\tfrac{1}{2} + 1 + \tfrac{1}{2}\right) = 1 \Rightarrow b = \tfrac{1}{\sqrt{2}}, a = \tfrac{1}{2}, c = \tfrac{1}{2}$$

$$|1\rangle_x = \tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{2}|-1\rangle$$

$$\frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0\hbar\begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{matrix} b = 0 \\ a + c = 0 \\ b = 0 \end{matrix}$$

$$|a|^2 + |b|^2 + |c|^2 = 1 \Rightarrow |a|^2(1 + 1) = 1 \Rightarrow a = \tfrac{1}{\sqrt{2}}, b = 0, c = -\tfrac{1}{\sqrt{2}}$$

$$|0\rangle_x = \tfrac{1}{\sqrt{2}}|1\rangle - \tfrac{1}{\sqrt{2}}|-1\rangle$$

$$\frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -1\hbar\begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{matrix} b = -a\sqrt{2} \\ a + c = -b\sqrt{2} \\ b = -c\sqrt{2} \end{matrix}$$

$$|a|^2 + |b|^2 + |c|^2 = 1 \Rightarrow |b|^2\left(\tfrac{1}{2} + 1 + \tfrac{1}{2}\right) = 1 \Rightarrow b = -\tfrac{1}{\sqrt{2}}, a = \tfrac{1}{2}, c = \tfrac{1}{2}$$

$$|-1\rangle_x = \tfrac{1}{2}|1\rangle - \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{2}|-1\rangle$$

(4) Initial state is $|\psi\rangle = |L_z = -\hbar\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Possible results of L_x measurement are eigenvalues of L_x: $L_x = \hbar, 0, -\hbar$ (or 1, 0, -1). The probabilities are

$$\mathcal{P}_{1x} = \left|{}_x\langle 1|\psi\rangle\right|^2 = \left|\left(\tfrac{1}{2}\langle 1| + \tfrac{1}{\sqrt{2}}\langle 0| + \tfrac{1}{2}\langle -1|\right)\left(|-1\rangle\right)\right|^2 = \left|\tfrac{1}{2}\right|^2 = \tfrac{1}{4}$$

$$\mathcal{P}_{0x} = \left|{}_x\langle 0|\psi_{in}\rangle\right|^2 = \left|\left(\tfrac{1}{\sqrt{2}}\langle 1| - \tfrac{1}{\sqrt{2}}\langle -1|\right)\left(|-1\rangle\right)\right|^2 = \left|-\tfrac{1}{\sqrt{2}}\right|^2 = \tfrac{1}{2}$$

$$\mathcal{P}_{-1x} = \left|{}_x\langle -1|\psi\rangle\right|^2 = \left|\left(\tfrac{1}{2}\langle 1| - \tfrac{1}{\sqrt{2}}\langle 0| + \tfrac{1}{2}\langle -1|\right)\left(|-1\rangle\right)\right|^2 = \left|\tfrac{1}{2}\right|^2 = \tfrac{1}{4}$$

The three probabilities add to unity, as they must.

(5) Initial state is $|\psi_{in}\rangle \doteq \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$. Possible results of L_z^2 measurement are eigenvalues of L_z^2.

L_z^2 is already diagonal:

$$L_z^2 \doteq \hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

so eigenvalues can be read off by inspection:

$$L_z^2 = \hbar^2, 0, \hbar^2 \quad \text{(or 1, 0, 1)}$$

Note the degeneracy: the states $|1\rangle$ and $|-1\rangle$ produce the same eigenvalue $\hbar^2$. Hence we must use the projection operator to find the state after a measurement that yields $\hbar^2$:

$$|\psi_{out}\rangle = \frac{P_{\hbar^2}|\psi_{in}\rangle}{\sqrt{\langle\psi_{in}|P_{\hbar^2}|\psi_{in}\rangle}}$$

For this case, we get

$$|\psi_{out}\rangle = \frac{(P_1 + P_{-1})|\psi_{in}\rangle}{\sqrt{\langle\psi_{in}|(P_1 + P_{-1})|\psi_{in}\rangle}} = \frac{(|1\rangle\langle 1| + |-1\rangle\langle -1|)|\psi_{in}\rangle}{\sqrt{\langle\psi_{in}|(|1\rangle\langle 1| + |-1\rangle\langle -1|)|\psi_{in}\rangle}}$$

$$(|1\rangle\langle 1| + |-1\rangle\langle -1|)|\psi_{in}\rangle \doteq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\langle\psi_{in}|(|1\rangle\langle 1| + |-1\rangle\langle -1|)|\psi_{in}\rangle = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{3}{4}$$

$$|\psi_{out}\rangle \doteq \frac{2}{\sqrt{3}} \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}$$

The probability is given by the expectation value of the projection, which is included in the above calculation

$$\mathcal{P}_{L_z^2=\hbar^2} = \langle\psi_{in}|(P_1 + P_{-1})|\psi_{in}\rangle = \langle\psi_{in}|(|1\rangle\langle 1| + |-1\rangle\langle -1|)|\psi_{in}\rangle = \frac{3}{4}$$

If we now measure L_z, then the possible results are the eigenvalues of L_z: $L_z = \hbar, 0, -\hbar$ (or 1, 0, -1). The probabilities are

$$\mathcal{P}_1 = \left|\langle 1|\psi_{out}\rangle\right|^2 = \left|\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}\right|^2 = \frac{1}{3}$$

$$\mathcal{P}_0 = \left|\langle 0|\psi_{out}\rangle\right|^2 = \left|\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}\right|^2 = 0$$

$$\mathcal{P}_{-1} = \left|\langle -1|\psi_{out}\rangle\right|^2 = \left|\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}\right|^2 = \frac{2}{3}$$

(6) If we know that

$$\mathcal{P}_1 = \left|\langle 1|\psi_{out}\rangle\right|^2 = \frac{1}{4}$$
$$\mathcal{P}_0 = \left|\langle 0|\psi_{out}\rangle\right|^2 = \frac{1}{2}$$
$$\mathcal{P}_{-1} = \left|\langle -1|\psi_{out}\rangle\right|^2 = \frac{1}{4}$$

Then we can solve these to find that

$$\langle 1|\psi_{out}\rangle = \tfrac{1}{2}e^{i\delta_1}$$
$$\langle 0|\psi_{out}\rangle = \tfrac{1}{\sqrt{2}}e^{i\delta_2}$$
$$\langle -1|\psi_{out}\rangle = \tfrac{1}{2}e^{i\delta_3}$$

noting that answers can be complex. Thus the initial state must be

$$|\psi_{in}\rangle = \tfrac{1}{2}e^{i\delta_1}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\delta_2}|0\rangle + \tfrac{1}{2}e^{i\delta_3}|-1\rangle$$

An overall phase is not physically measurable, but relative phases are. For example, if we calculate

$$\begin{aligned}
\mathcal{P}_{1x} &= \left|{}_x\langle 1|\psi_{in}\rangle\right|^2 = \left|\left(\tfrac{1}{2}\langle 1| + \tfrac{1}{\sqrt{2}}\langle 0| + \tfrac{1}{2}\langle -1|\right)\left(\tfrac{1}{2}e^{i\delta_1}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\delta_2}|0\rangle + \tfrac{1}{2}e^{i\delta_3}|-1\rangle\right)\right|^2 \\
&= \left|\left(\tfrac{1}{4}e^{i\delta_1} + \tfrac{1}{2}e^{i\delta_2} + \tfrac{1}{4}e^{i\delta_3}\right)\right|^2 = \left|e^{i\delta_1}\left(\tfrac{1}{4} + \tfrac{1}{2}e^{i(\delta_2-\delta_1)} + \tfrac{1}{4}e^{i(\delta_3-\delta_1)}\right)\right|^2 \\
&= \left(\tfrac{1}{4} + \tfrac{1}{2}e^{-i(\delta_2-\delta_1)} + \tfrac{1}{4}e^{-i(\delta_3-\delta_1)}\right)\left(\tfrac{1}{4} + \tfrac{1}{2}e^{i(\delta_2-\delta_1)} + \tfrac{1}{4}e^{i(\delta_3-\delta_1)}\right) \\
&= \tfrac{1}{16} + \tfrac{1}{4} + \tfrac{1}{16} + \tfrac{1}{4}\cos(\delta_2-\delta_1) + \tfrac{1}{8}\cos(\delta_3-\delta_1) + \tfrac{1}{4}\cos(\delta_3-\delta_2)
\end{aligned}$$

we can rewrite this in terms of two phases:

$$\phi_1 = \delta_2 - \delta_1$$
$$\phi_2 = \delta_3 - \delta_1$$
$$\mathcal{P}_{1x} = \tfrac{1}{16} + \tfrac{1}{4} + \tfrac{1}{16} + \tfrac{1}{4}\cos\phi_1 + \tfrac{1}{8}\cos\phi_2 + \tfrac{1}{4}\cos(\phi_2 - \phi_1)$$

so we can safely set one phase to zero and write

$$|\psi_{in}\rangle = \tfrac{1}{2}|1\rangle + \tfrac{1}{\sqrt{2}}e^{i\phi_1}|0\rangle + \tfrac{1}{2}e^{i\phi_2}|-1\rangle$$

2.23 (a) The commutator is

$$[A,B] = AB - BA \doteq \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}\begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix} - \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}\begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}$$

$$\doteq \begin{pmatrix} a_1b_1 & 0 & 0 \\ 0 & 0 & a_2b_2 \\ 0 & a_3b_2 & 0 \end{pmatrix} - \begin{pmatrix} a_1b_1 & 0 & 0 \\ 0 & 0 & a_3b_2 \\ 0 & a_2b_2 & 0 \end{pmatrix}$$

$$\doteq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_2(a_2 - a_3) \\ 0 & b_2(a_3 - a_2) & 0 \end{pmatrix} \neq 0$$

so they do not commute.

(b) A is already diagonal, so the eigenvalues and eigenvectors are obtained by inspection. The eigenvalues are

$$a_1, a_2, a_3$$

and the eigenvectors are

$$|a_1\rangle = |1\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |a_2\rangle = |2\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |a_3\rangle = |3\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

For B, diagonalization yields the eigenvalues

$$\begin{vmatrix} b_1 - \lambda & 0 & 0 \\ 0 & -\lambda & b_2 \\ 0 & b_2 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad (b_1 - \lambda)(\lambda^2 - b_2^2) = 0$$

$$\Rightarrow \quad \lambda = b_1, b_2, -b_2$$

and the eigenvectors

$$\begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = b_1 \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \Rightarrow \quad \begin{matrix} b_1 u = b_1 u \\ b_2 w = b_1 v \\ b_2 v = b_1 w \end{matrix} \quad \Rightarrow \quad w = v = 0$$

$$|u|^2 + |v|^2 + |w|^2 = 1 \quad \Rightarrow \quad |u|^2 = 1 \quad \Rightarrow \quad u = 1, v = 0, w = 0 \quad \Rightarrow \quad |b_1\rangle = |1\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = b_2 \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \Rightarrow \quad \begin{matrix} b_1 u = b_2 u \\ b_2 w = b_2 v \\ b_2 v = b_2 w \end{matrix} \quad \Rightarrow \quad u = 0,\ w = v$$

$$\langle b_2 | b_2 \rangle = 1 \quad \Rightarrow \quad |v|^2 + |w|^2 = 1 \quad \Rightarrow \quad u = 0, v = \tfrac{1}{\sqrt{2}}, w = \tfrac{1}{\sqrt{2}} \quad \Rightarrow \quad |b_2\rangle = \tfrac{1}{\sqrt{2}}\big(|2\rangle + |3\rangle\big) \doteq \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = -b_2 \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \Rightarrow \quad \begin{matrix} b_1 u = -b_2 u \\ b_2 w = -b_2 v \\ b_2 v = -b_2 w \end{matrix} \quad \Rightarrow \quad u = 0,\ w = -v$$

$$\langle -b_2 | -b_2 \rangle = 1 \quad \Rightarrow \quad |v|^2 + |w|^2 = 1 \quad \Rightarrow \quad u = 0, v = \tfrac{1}{\sqrt{2}}, w = -\tfrac{1}{\sqrt{2}} \quad \Rightarrow \quad |-b_2\rangle = \tfrac{1}{\sqrt{2}}\big(|2\rangle + |3\rangle\big) \doteq \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

c) If B is measured, the possible results are the allowed eigenvalues $b_1, b_2, -b_2$. If the initial state is $|\psi_i\rangle = |2\rangle$, then the probabilities are

$$\mathcal{P}_{b_1} = \left|\langle b_1 | \psi_i \rangle\right|^2 = \left|\langle 1 | 2 \rangle\right|^2 = 0$$

$$\mathcal{P}_{b_2} = \left|\langle b_2 | \psi_i \rangle\right|^2 = \left|\tfrac{1}{\sqrt{2}}\big(\langle 2| + \langle 3|\big)|2\rangle\right|^2 = \tfrac{1}{2}$$

$$\mathcal{P}_{-b_2} = \left|\langle -b_2 | \psi_i \rangle\right|^2 = \left|\tfrac{1}{\sqrt{2}}\big(\langle 2| - \langle 3|\big)|2\rangle\right|^2 = \tfrac{1}{2}$$

If A is then measured, the possible results are the allowed eigenvalues a_1, a_2, a_3. If b_2 was the first result, then the new state is $|b_2\rangle$ and when A is measured the subsequent probabilities are

$$\mathcal{P}_{a_1} = \left|\langle a_1 | b_2 \rangle\right|^2 = \left|\langle 1 | \tfrac{1}{\sqrt{2}}\left(|2\rangle + |3\rangle\right)\right|^2 = 0$$
$$\mathcal{P}_{a_2} = \left|\langle a_2 | b_2 \rangle\right|^2 = \left|\langle 2 | \tfrac{1}{\sqrt{2}}\left(|2\rangle + |3\rangle\right)\right|^2 = \tfrac{1}{2}$$
$$\mathcal{P}_{a_3} = \left|\langle a_3 | b_2 \rangle\right|^2 = \left|\langle 3 | \tfrac{1}{\sqrt{2}}\left(|2\rangle + |3\rangle\right)\right|^2 = \tfrac{1}{2}$$

If $-b_2$ was the first result, then the new state is $|-b_2\rangle$ and when A is measured the subsequent probabilities are

$$\mathcal{P}_{a_1} = \left|\langle a_1 | -b_2 \rangle\right|^2 = \left|\langle 1 | \tfrac{1}{\sqrt{2}}\left(|2\rangle - |3\rangle\right)\right|^2 = 0$$
$$\mathcal{P}_{a_2} = \left|\langle a_2 | -b_2 \rangle\right|^2 = \left|\langle 2 | \tfrac{1}{\sqrt{2}}\left(|2\rangle - |3\rangle\right)\right|^2 = \tfrac{1}{2}$$
$$\mathcal{P}_{a_3} = \left|\langle a_3 | -b_2 \rangle\right|^2 = \left|\langle 3 | \tfrac{1}{\sqrt{2}}\left(|2\rangle - |3\rangle\right)\right|^2 = \tfrac{1}{2}$$

d) If two operators do not commute, then the corresponding observables cannot be measured simultaneously. Part (a) tells us that the operators A and B not commute. Part (c) tells us that measurement B "disturbs" the measurement of A so the two measurements are not compatible (cannot be made simultaneously). So even though we started in state $|\psi_i\rangle = |2\rangle$, which is an eigenstate of A (meaning we know that the system has $A = a_2$), the measurement of B puts the system into a state for which A is now not well defined, as evidenced by the subsequent A measurement.