PH 652 Quantum Mechanics 11 February 2016

Homework 4

Due Wednesday 24 February

1. 12.3.4, p. 315 from Shankar

2. 12.3.6, p. 316 from Shankar

3. 12.5.2, p. 329 from Shankar. For the three cases: $j = ½$, $j = 1$, and $j = 3/2$, write down all 6 matrices representing J^2, J_x, J_y, J_z, J_+, and J_-.

4. 12.5.13, p. 338 from Shankar

5. Consider a system described by the Hamiltonian

$$H = \varepsilon_\alpha a^\dagger a + \varepsilon_\beta b^\dagger b + g\left(a^\dagger b + b^\dagger a\right)$$

where the operators a and b satisfy the relations

$$a^\dagger a + aa^\dagger = b^\dagger b + bb^\dagger = 1$$
$$aa = bb = 0$$
$$\left[a, b\right] = \left[a, b^\dagger\right] = 0$$

The operators $N_\alpha = a^\dagger a$, $N_\beta = b^\dagger b$, and $N = N_\alpha + N_\beta$ are observables, and N_α and N_β constitute a complete set of commuting operators.

a) Using the operator equations above, find the eigenvalues of N_α .

b) Show that N is a constant of the motion.

c) Find the energy eigenvalues of the system.