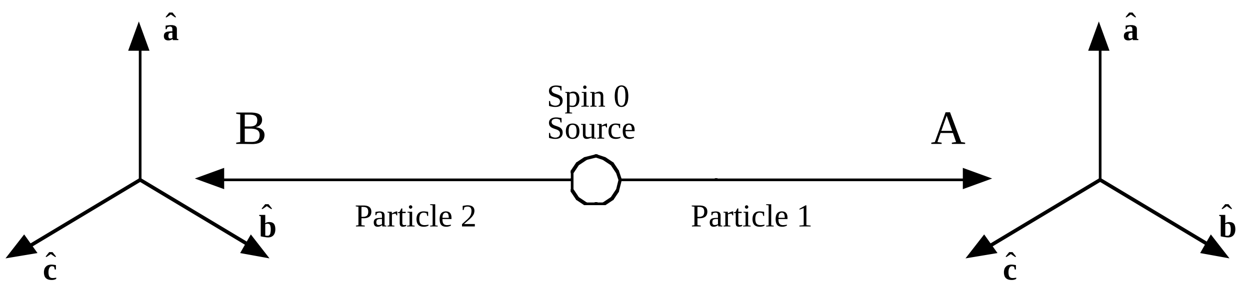
Einstein-Podolsky-Rosen Paradox

The Einstein-Podolsky-Rosen Paradox is a gedanken experiment designed to show that quantum mechanics is an incomplete description of reality. The authors were uncomfortable with the quantum mechanical notion that we can only know certain properties of an atom (*e.g.*, only one of the spin components, not all three). The gedanken experiment attempts to prove that the unknown properties are really there (they are *elements of reality* in the authors' words).

The experimental situation is depicted below (this version of the EPR experiment is due to David Bohm). A spin 0 source decays into two spin 1/2 particles, which by conservation of angular momentum must have opposite spin projections and by conservation of momentum must head in opposite directions. Observers *A* and *B* are on opposite sides of the source and each has a Stern-Gerlach apparatus to measure the spin projection of the particle headed in its direction. Whenever one observer measures spin up along a given direction, then the other observer measures spin down along that same direction. The quantum state of the two particle system can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right],$$

where the subscripts label the particles and the relative minus sign ensures that this is a spin-0 state. Because of the correlation between the measurements, if observer *A* measures spin up along a given direction, for example $S_z = +\hbar/2$, then we can predict with 100% certainty what the result of observer *B*'s measurement will be ($S_z = -\hbar/2$), without performing the measurement or disturbing particle 2 in any way. EPR contend that if we can predict a measurement result with 100% certainty, then that result must be a "real" property of the particle -- it must be an element of reality. Since the particles are widely separated, this element of reality must be independent of what observer *A* does, and hence must have existed all along. Observer *A* could have chosen to measure S_x or S_y, which by the same reasoning means that S_x and S_y for particle 2 must also be



elements of reality. Quantum mechanics maintains that we can only know one spin component at a time for a single particle. Since it thus does not describe all the elements of reality of the particle, quantum mechanics must be an incomplete description of physical reality.

If EPR are correct, then the elements of reality, which are often called hidden variables or instructions sets, are really there but for some reason we cannot know all of them at once. Thus one can imagine that there are different types of particles with different instructions sets that determine the results of measurements. One can also imagine that the populations or probabilities of these different sets can be properly adjusted in a hidden variable theory to produce results consistent with quantum mechanics. Since quantum mechanics and a hidden variable theory cannot be distinguished by experiment, the question of which is correct is then left to the realm of metaphysics. For many years, this was what many physicists believed.

In 1964, John Bell showed that there are specific measurements that can be made to distinguish between a hidden variable theory and quantum mechanics. By considering measurements that observers *A* and *B* make along three different directions (all in a plane as shown above), he derived an inequality that could be tested by experiment. He derived a very general relation, but we will deal with a specific one here to make life easy.

Consider three directions in a plane as shown, each 120° from any of the other two. Each observer makes measurements of the spin projection along one of these three directions, chosen randomly. Any single result can only be + or -, and we calculate the probability that the results from a correlated pair (*i.e.*, one decay from the source) are the same (++ or --) or opposite (+– or –+), where +-, for example, means observer *A* recorded a + and observer *B* recorder a -. We know that when both observers measure along the same direction, then only a +- or a -+ is possible. To reproduce this aspect of the data, a hidden variable theory would need 8 instruction sets as shown in the table. We don’t yet know what the probabilities are for cases where the observers do not measure along the same directions, so we do not assign any populations (or weights or probabilities) to the different instruction sets. Presumably we can adjust these as needed to make sure that the hidden variable theory agrees with the actual (or quantum mechanical results).

Now use the instruction sets to calculate the probability that the results are the same ($\mathcal{P}_{same} = \mathcal{P}_{++} + \mathcal{P}_{--}$) and the probability that the results are opposite ($\mathcal{P}_{opp} = \mathcal{P}_{+-} + \mathcal{P}_{-+}$), considering all possible measurements. There are 9 different combinations of measurement directions for the pair of observers. If we consider particles of type 1, then for each of these 9 possibilities, the results will be opposite (+-). The results can never be the same. The same argument holds for type 8 particles. For type 2 particles, there will be 4 possibilities of recording the same results and

Instruction Sets (Hidden Variables)

Population	Particle 1	Particle 2
N_1	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}+,\hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}-,\hat{\mathbf{c}}-)$
N_2	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}+,\hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}-,\hat{\mathbf{c}}+)$
N_3	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}-,\hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}+,\hat{\mathbf{c}}-)$
N_4	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}-,\hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}+,\hat{\mathbf{c}}+)$
N_5	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}+,\hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}-,\hat{\mathbf{c}}-)$
N_6	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}+,\hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}-,\hat{\mathbf{c}}+)$
N_7	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}-,\hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}+,\hat{\mathbf{c}}-)$
N_8	$(\hat{\mathbf{a}}-,\hat{\mathbf{b}}-,\hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+,\hat{\mathbf{b}}+,\hat{\mathbf{c}}+)$

5 possibilities for recording opposite results. We thus arrive at the following probabilities for the different particle types:

$$\left.\begin{aligned}\mathcal{P}_{opp} &= 1\\ \mathcal{P}_{same} &= 0\end{aligned}\right\}\ \text{types } 1\,\&\,8 \qquad\qquad \left.\begin{aligned}\mathcal{P}_{opp} &= \frac{5}{9}\\ \mathcal{P}_{same} &= \frac{4}{9}\end{aligned}\right\}\ \text{types } 2 \rightarrow 7$$

Now average over all the possible particle types to find the probabilities of recording the same or opposite results in all the measurements. The probability of any particular particle type, for example type 1, is simply $\mathrm{N}_1 / \sum N_i$ (recall we will adjust the actual values later as needed). Thus the averaged probabilities are:

$$\mathcal{P}_{opp} = \frac{1}{\sum_i N_i}\left(N_1 + N_8 + \frac{5}{9}\left(N_2 + N_3 + N_4 + N_5 + N_6 + N_7\right)\right) \geq \frac{5}{9},$$

$$\mathcal{P}_{same} = \frac{1}{\sum_i N_i}\frac{4}{9}\left(N_2 + N_3 + N_4 + N_5 + N_6 + N_7\right) \leq \frac{4}{9},$$

where the inequalities follow simply because the sum of all the probabilities for the particular particle types must sum to one. In summary, we can adjust the populations all we want but that will always produce probabilities of the same or opposite measurements that are bound by the above inequalities. That is what is meant by a Bell inequality.

What does quantum mechanics predict for these probabilities? For this simple system of 2 spin 1/2 particles, we can easily calculate them. Assume that observer *A* records a "+" along some direction (of the three) and define that direction as the *z*-axis (no law against that). Then we know that the quantum state of particle 2 is $|-\rangle$. The probability that observer *B* records a "+" along a direction at some angle θ with respect to the *z*-axis is

$$\mathcal{P}_{same} = \left|{}_n\langle+|-\rangle\right|^2 = \left|\left\{\cos\frac{\theta}{2}\langle+| + e^{-i\phi}\sin\frac{\theta}{2}\langle-|\right\}|-\rangle\right|^2 = \sin^2\frac{\theta}{2},$$

where $|+\rangle_n$ is the eigenstate for spin up along the direction of measurement. The same result will be obtained if you assume *A* records a "-" and ask for the probability that *B* records a "-" also. The probability that observer *B* records a "-" along this direction, when *A* records a "+" , (hence opposite results) is

$$\mathcal{P}_{opp} = \left|{}_n\langle-|-\rangle\right|^2 = \left|\left\{\sin\frac{\theta}{2}\langle+| - e^{-i\phi}\cos\frac{\theta}{2}\langle-|\right\}|-\rangle\right|^2 = \cos^2\frac{\theta}{2}.$$

Since the angle θ will be 0° in 1/3 of the measurements and 120° in 2/3 of the measurements, the average probabilities will be

$$\mathcal{P}_{opp} = \frac{1}{3}\cdot\cos^2\frac{0°}{2} + \frac{2}{3}\cdot\cos^2\frac{120°}{2} = \frac{1}{3}\cdot 1 + \frac{2}{3}\cdot\frac{1}{4} = \frac{1}{2},$$

$$\mathcal{P}_{same} = \frac{1}{3}\cdot\sin^2\frac{0°}{2} + \frac{2}{3}\cdot\sin^2\frac{120°}{2} = \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot\frac{3}{4} = \frac{1}{2}.$$

These predictions of quantum mechanics are inconsistent with the inequalities derived from hidden variable theories. Since these probabilities can be measured, we can do experiments to test whether hidden variable theories are possible. The results agree with quantum mechanics and hence exclude the possibility of hidden variable theories.

The EPR paradox also raises issues regarding collapse of the quantum state and how a measurement by *A* can instantaneously alter the quantum state at *B*. However, there is no information transmitted instantaneously and so no violation of relativity. What observer *B* measures is not affected by any measurements that *A* makes. They only notice that when they get together and compare results that some of the measurements (along the same axes) are correlated.

Schrödinger Cat Paradox

The Schrödinger cat paradox is a gedanken experiment designed by Schrödinger to illustrate some of the problems of quantum measurement, particularly in the extension of quantum mechanics to classical systems. The experimental apparatus consists of a radioactive atom, a Geiger counter, a hammer, a bottle of cyanide gas, a cat, and a box. The atom has a 50% probability of decaying in one hour. The components are put together such that if the atom decays, it triggers the counter, which causes the hammer to break the bottle and release the poisonous gas, killing the cat. Thus, after one hour there is 50% probability that the cat is dead.

We can describe the quantum state of the atom as

$$\left|\psi_{atom}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\psi_{undecayed}\right\rangle + \left|\psi_{decayed}\right\rangle\right].$$

The apparatus has been designed such that there is a one-to-one correspondence between the undecayed atomic state and the live-cat state and between the decayed atomic state and the dead-cat state. Though the cat is macroscopic, it is made up of microscopic particles and so should be describable by a quantum state (albeit a complicated one). Thus we expect that the quantum state of the cat after one hour is

$$\left|\psi_{cat}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\psi_{alive}\right\rangle + \left|\psi_{dead}\right\rangle\right].$$

Both quantum calculations and classical reasoning would predict 50/50 probabilities of observing an alive or dead cat when we open the box. However, quantum mechanics would lead us to believe that the cat was neither dead nor alive before we opened the box, but rather was in a superposition of states and the quantum state only becomes the single alive or dead state when we open the box and make the measurement by observing the cat. But our classical experiences clearly run counter to this. We would say that the cat really was dead or alive, we just did not know it yet. (Imagine that the cat is wearing a cyanide sensitive watch -- the time will tell us when the cat was killed if it is dead!)

The main issues raised by this thought experiment are (1) Can we describe macroscopic states quantum mechanically? and (2) What causes the collapse of the wave function?

The Copenhagen or standard interpretation of quantum mechanics championed by Bohr and Heisenberg maintains that there is a boundary between the classical and quantum worlds. We describe microscopic systems (the atom) with quantum states and macroscopic systems (the cat, or even the Geiger counter) with classical rules. The measurement apparatus causes the quantum state to collapse and yield the single classical or meter result. Where to draw this line is not clear and will depend on the problem. Others have argued that the human consciousness is responsible for collapsing the wave function, while some have argued that there is no collapse, just bifurcation into alternate, independent universes. Since most of these points of view are untestable, it is often left as a philosophical debate.

More recent discussions and experiments have focused on the issue of using quantum mechanics to describe macroscopic systems. By studying mesoscopic systems that are small enough to control precisely, yet large enough to have macroscopically distinguishable states, one can probe the region between the quantum and the classical worlds. In recent experiments in quantum optics, it has been shown that the relative phase between two parts of a superposition state becomes randomized very quickly, yielding a mixture state, which is not distinguishable from a classical probability mixture. It has been shown that this coherence decay proceeds more quickly as the size of the system (and hence its complexity) becomes larger. It may not be long before we conclude that the wave function collapse is not distinct from ordinary Schrödinger time evolution, but rather just a consequence of the decoherence of large multi-component states.