Arrows as Anchors: Conceptual Blending and Student Use of Electric Field Vector Arrows

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We use the theory of conceptual blending with material anchors to describe how people make meaning of the vector arrows representation of electric fields. We describe this representation as a conceptual blend of a spatial (coordinate) input space and an electric-field-as-arrows space (which itself is a blend of electric field concept with arrows). This representation possesses material features including the use of spatial extent (e.g., distance on paper) to represent the coordinate space and to represent the magnitude of electric field vectors. As a result, this representation supports a geometric interpretation of the electric field, breaking the field into components, and the addition of two fields at a point. The material features also emphasize the spatial relationships between the source(s) and points where the field is represented. However, the material features also necessitate sampling and do not generally support the rapid superposition of two fields at all points. We illustrate this analysis with examples from clinical problem-solving interviews with upper-division physics majors, and interpret students' errors in using this representation as resulting from conflict between the input spaces in the blend.

Keywords: Conceptual Blending, Material Anchors, Electric Field Vectors, Representational Fluency
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INTRODUCTION

The ability to represent physical phenomena using various representations is one of the hallmarks of expertise in physics [1], but students often have difficulty using these disciplinary representations [2, 3].

We apply conceptual blending theory [4] to the use of these representations, specifically focusing on how the material features of the blended space might influence (not always positively) the reasoning of learners. Conceptual blending theory has previously been used to describe the relationship between mathematics and physics during problem solving [5] and how students construct conceptual explanations for physical phenomena [6].

The goals of the paper are to (1) demonstrate how conceptual blending theory can be used to explain how people make meaning of external representations in physics through the specific example of the vector arrows representation of electric field, and (2) discuss some of the roles of material anchors that are consistent with students' use of representations during a superposition of two electric fields task.

First, we briefly discuss the elements of conceptual blending theory relevant for our analysis. Second, we discuss how the vector arrows representation of electric fields constitutes a conceptual blend. Third, we discuss how the material features of the representation support student performance in a superposition task.

CONCEPTUAL BLENDING THEORY WITH MATERIAL ANCHORS

Conceptual blending is a theoretical framework aimed at describing how people make meaning by selectively projecting elements of separate mental *input* spaces onto a *blended* space from which new meaning emerges. Input mental spaces are connected through *vital conceptual relations*, such as time, space, identity, analogy, etc. The elements and organizing frames of the input spaces can be selectively projected into the blend in a variety of combinations. For example, in a *double scope blend*, both input spaces contribute elements of their organizing frames to the blended space. We will demonstrate that the vector arrows representation of electric fields is a double scope blend with a conflict on the vital conceptual relation of space. The conflict may lead to student difficulties in using the vectors arrows representation for electric field.

Sometimes a conceptual blend involves a material object (or a collection of objects) called a "material anchor". While Fauconnier & Turner [4] emphasize the "sign" aspects of material anchors, such as bills or coins in the case of money or word forms in the case of writing, Hutchins [7] emphasizes the stabilizing function of material anchors to support cognition. For example, a line of people and an ordering [8] can be combined to form a blended space corresponding to a

queue. In this case, the material anchor is the line of people. The existence of this anchor provides elements of the queue (i.e. ordered individuals) so that other operations can be performed (like counting the number of people who are waiting for service or determining who will be the fourth person to receive service). The blend arises from the combination of the concept (sequential line) with the material anchor. Without the material anchor of the line of people (i.e. if the people were haphazardly milling about), or without the conceptual structure (the sequence), these operations would be more difficult to accomplish.

THE ELECTRIC FIELD VECTOR MAP CONCEPTUAL BLEND

A vector-arrows representation of an electric field over a region of space is a conceptual blend we call the *Electric Field Arrows Map* (Figure 1). One input space for this blend is the *Electric Field Arrows* space (which is itself a blend of an Electric Field Concept space with an Arrows space). In the Electric Field Arrows space, the length of the arrow corresponds to the strength of the field, and the orientation of the arrow corresponds to the direction of the electric field. The material anchor of this space is the arrow. The organizing frame of this Electric Field Arrows space is a vectors frame, where the important features are specifically related to properties of vectors, namely, the magnitude (amount) and the direction.

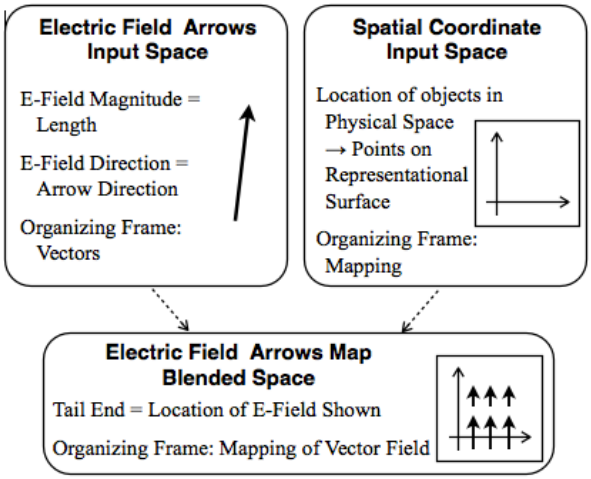

FIGURE 1. Network map for the double scope Electric Field Arrow Map blend.

The other input space for the Electric Field Arrows Map blend is a *Spatial Coordinate* space, where physical space is mapped onto a representational surface. In this Spatial Coordinate Space, a subset of points in space corresponds to locations on the representational surface. Sometimes the objects in real space, like a charge distribution, are represented. A coordinate system could also be imposed. In this blend, the material anchor is the representational surface, and the organizing frame is a mapping frame.

In the Electric Field Arrows Map blend, these two input spaces are combined to represent the electric field in the physical space. The physical space is mapped onto the representational space, and the electric field in the physical space is represented on the representational surface with arrows.

This blend is a double-scope blend. Both the mapping frame from the Spatial Coordinate space and the vector space frame from the Electric Field Arrows space are necessary for correctly interpreting the blend. When the charged sources are included in the mapping, this blend emphasizes the spatial relationships between the sources of electric field and the location where the field is represented.

A conflict occurs in this blend for the vital conceptual relation of space. In the Spatial Coordinate space, the points in space correspond to locations of electric field and charged objects. In the Electric Field Vector Arrows space, the points of space lying along the length of an arrow collectively correspond to the magnitude of the electric field. In the blend, the element of space maps onto more than one meaning: the *location* of where the electric field is and the *magnitude* of the electric field. In the blend, distance is ambiguous; distance in the representational space can represent either physical lengths or electric field strengths.

A consequence of this dual role of space is the need for sampling. One cannot draw an arrow at every point in space even though the electric field exists at every point. The arrows would overlap and be unintelligible (not to mention, it would take an inconveniently long time to draw). Therefore, typically only a subset of points is chosen, along with an arbitrary global scaling factor, so that the arrows do not overlap. Other consequences include the ability to visually compare field strength at different locations, add vectors geometrically, and visualize patterns/symmetry.

STUDENT REASONING WITH THE ELECTRIC FIELD ARROWS MAP

Methods

We conducted n=8 exploratory interviews with advanced undergraduate physics majors at Oregon State University who participated in the Paradigms in Physics program [9]. The interviews were semi-structured in nature and students were asked to think aloud as they were performing the interview tasks. The full interviews lasted about an hour, and included tasks

about electric fields and quantum measurements. This paper, however, focuses only on discussion related to electric fields, about half of the interview for most students. The interviews were video taped and transcribed for analysis.

Students were first asked to represent the electric field due to an infinite charged sheet in as many different ways as they could. After the students exhausted their own suggestions for representations, the interviewer suggested additional representations, including vector arrows, field lines, equations, and graphs of the electric field component vs. spatial coordinate to ensure that all of these representations were discussed. During this part of the interview, it was established (by either the student or the interviewer) that the electric field due to a charged infinite sheet is uniform and perpendicular to the sheet (away from the sheet in the case of a positive charge density).

Next, the students were asked to consider two charged infinite sheets intersecting at a 45° angle, with one sheet in the z=0 plane. The students were then asked how they would represent the electric field due to the sheets. For this task, the students must use superposition to add the electric field from each sheet. All of the students assumed that the sheets were positively charged, uniformly charged and had equal charge densities.

Our analysis focuses on the students' use of electric field vector arrows during this second task of representing the electric field from the two sheets. Specifically, we looked for patterns of the students' use of the electric field vector arrows, guided by the question: What role(s) might the material anchors of the Electric Field Arrows Map blend play in students' solutions of this task?

The Stability of the Material Anchors Supports Comparison and Extrapolation

When generating an algebraic representation of the combined electric field, nearly all the students used the spatial/geometric features of the Electric Field Arrow Map to determine the components of the electric field vectors by comparing an electric field vector arrow from each sheet with the axes of the chosen coordinate system. Some students also drew the separate components of the electric field vector arrow and used trigonometry to determine the lengths/magnitudes of the components. The stability of the drawn arrows and the drawn coordinate system supports this comparison and computation.

The students generally began representing the combined electric field by drawing electric field vector arrows due to each sheet. One might expect that the students would have picked a field point and drawn the electric field vector arrows with their tails located at the field point (like a free-body diagram). Instead, we found that students generally first drew arrows with their tails near or even touching the charged sheets (Figure 2), and then extrapolated these initial arrows to other points in the region. It seems that these students first established the direction of the field at locations near/on the sources, perhaps to distinguish the electric field contributions from each sheet (because of the arrows' proximity to their own sources and large separation from other sources). These initial arrows may then provide stability to support the drawing of other arrows farther away from the sources, where the association with the sources are possibly more tenuous for the student.

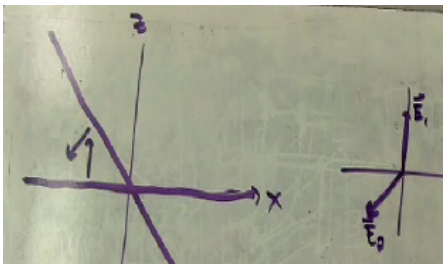

FIGURE 2. An example of a student first drawing electric field arrows drawn near/on the lines representing the charged sheets (left), and then placing the electric field arrows tail to tail to find the combined electric field at a point (right).

The Dual Role of Space

One student, Jim, drew a series of vector arrows in a region defined by the two intersecting sheets. At the points near the intersection, he drew short vector arrows, but at points farther away from the intersection, he drew longer vector arrows (Figure 3). This result is inconsistent both with idea that the field from each sheet is uniform (which Jim correctly articulated in the first part of the interview) and with the idea that there would be a larger electric field at points closest to the locations of the charges (a general idea consistent with Coulomb's Law for a point charge).

However, Jim's result is consistent with the incorrect idea that the electric field exists along the length of the arrow. This misconception about electric field vector arrows has been previously documented in the context of calculating flux [10]. In particular, Jim drew small arrows near the intersection of the sheets where the region itself is small and presumably so that the arrows do not cross the boundary formed by the charged sheet.

We propose that this behavior may result from the conflict between the input spaces of the blend on the

vital relation of space. In the normative version of the blend, the points of space lying along the arrow do not correspond to locations of the field. However, for students like Jim, the points lying along the length of the arrow may correspond not only to the magnitude of the field, but also to the locations of the electric field (rather than the location being only at the tail of the arrow). As a result, the sheets (artificially) confine the electric field arrows so that the arrows to not "carry" electric field into a region where the electric field is different.

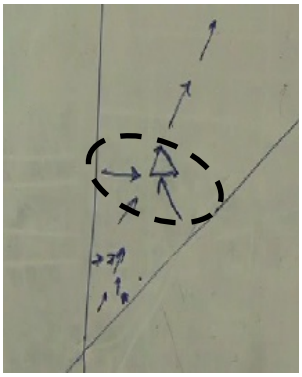

FIGURE 3. Jim's Electric Field Arrow Map for two charged infinite sheets. The long lines represent the charged sheets. Jim first drew the arrows (that we have circled) representing the electric fields from each sheet and then he added them head to tail as shown. Then he drew the other arrows (outside our circle) representing the combined electric field due to the two sheets. The smallest arrows lie closest to the intersection, and the arrows nearest the sheets are perpendicular to the sheets.

Electric Field Vector Map Highlights Distances Between Source and Field Points

An advantage of the Electric Field Arrow Map is that the distance between the source and the field point is easily discerned because the map emphasizes spatial relationships. This is generally advantageous because the magnitude of the electric field due to a point charge is inversely proportional to the square of this distance, and it is usually the case that the closer the field point is to the sources, the greater their affect on the electric field.

However, for this interview task, where the sheets are infinite and the electric field in each region is uniform, attending to this distance may not be productive. This material feature (emphasis of spatial relationships) may support some students' error of taking into account this distance even though the uniform nature of the electric field from each sheet was previously established in the interview. Like Jim (Figure 2), two others who made this error drew perpendicular arrows very near each sheet.

Conclusion

We have demonstrated that conceptual blending theory with material anchors provides an explanatory framework consistent with students' use of external representations. We have identified the vectors arrows representation of and electric fields in a region of space as a conceptual blend, and have illustrated how the material features of this representation support student reasoning about the superposition of electric fields.

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