Tangible Metaphors

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Abstract: Upper-division physics requires students to use abstract mathematical objects to model measurable properties of physical entities. We have developed activities that engage students in using their own bodies or simple home-built apparatus as metaphors for novel (to the students) types of mathematical objects. These tangible metaphors are chosen to be rich, robust, and flexible so that students can explore several properties of the mathematical objects over an extended period of time. The collaborative nature of the activities and inherent silliness of "dancing" out the behavior of currents or spin ½ states certainly increases the fun in the classroom and may also decrease students' fear of learning about these mathematical objects. We include examples from the electromagnetism, quantum mechanics, and thermodynamics content in the Paradigms in Physics program at Oregon State University.

Keywords: Upper-division physics, embodied cognition, kinesthetic activity, conceptual blend, material anchor.
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INTRODUCTION

One of the essential characteristics of physics problem solving is the process of modeling a physics situation with mathematical structures that have appropriate geometric and algebraic properties.[1] From the point of view of conceptual blending theory [1], the mapping between the physics situation and the mathematical structures becomes, for the expert, a *conceptual blend* [2]. "Conceptual blending is a basic mental operation that leads to new meaning, global insight, and conceptual compressions useful for memory and manipulation of otherwise diffuse ranges of meaning" [3].

At the middle-division level, when students are first transitioning from their introductory courses to the upper-division physics courses, they are confronted with a fire hose of new concepts: both new physics situations (*e.g.* charge and current distributed throughout space in different ways, spin ½ states for electrons, and mechanisms for getting energy into and out of thermodynamic systems) and new mathematical structures (*e.g.* scalar and vector fields in various dimensions, elements CP_2 (the Bloch sphere), and a wild variety of partials derivatives related by obscure algebraic relations). As both the physical and mathematical sides of the process become both more complicated and more abstract, the necessary conceptual blends become less intuitive and harder to achieve. An important pedagogical challenge for teachers is to facilitate this process appropriately.

Physics education researchers have long understood the special value of kinesthetic activities (see the lovely reminiscence by Arnold Arons [4] and an early activity from PSSC [5]). At Oregon State University, we have developed a number of kinesthetic activities [6]. A recent surge of interest in these activities at the collegiate level has been sweeping the physics education community, not only interest in developing such activities (Energy Theater [7] is a particularly lovely example) but also an interest in developing a theoretical perspective for analyzing them [7-10].

One way of understanding the pedagogical value of kinesthetic activities is through the theoretical perspective of embodied cognition. According to this perspective, "we understand the kinds of things that may exist in the world (ontology) in terms of sensory-motor experiences such as object permanence and movement, and that we express this understanding linguistically, through metaphor" [7, 11] Therefore, kinesthetic activities may be particularly powerful because they appeal to this fundamental way of understanding the physical universe.

Similarly, the framework of distributed cognition also gives insight to the pedagogical value of kinesthetic activities. According to this perspective, cognition is not limited to an individual but may be distributed across members of a social group and coordinated with external material and environmental structures [12]. Traditionally in physics, these external structures usually take the form of equations, graphs, and diagrams. In this view, external representations, such as equations, graphs, diagrams, as well as manipulatives and bodies involved in kinesthetic

[1]For the purposes of this paper, we will largely ignore the fascinating question of whether "a physics situation" as described in problem solving is itself only a model of a more fundamental reality based on our current understanding of physical laws.

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activities, are artifacts with both material (physical) and ideal (conceptual) elements [13]. The conceptual features of a representation are the meanings or ideas that are conveyed, whereas material features are physical aspects related to the representation's construction. Material features are particularly relevant in kinesthetic activities in that they can transform how tasks are accomplished with the representation. For example, the procedures for calculating a derivative using an equation and using a graph are much different. As Hutchins puts it, "each tool presents the task to the user as a different sort of cognitive problem requiring a different set of cognitive abilities or a different organization of the same set of abilities" [12, p. 154]. Therefore, material features are important in the pedagogical affordances of a representation.

We will use the term *tangible metaphor* to refer to the deliberate use of a representation which can be explored kinesthetically (either because the individual's own body is part of the representation or because the representation can be physically touched) to mediate understanding of the relationship between a physics situation and the associated mathematical structures. We are particularly interested in the classroom use of tangible metaphors where we have found the social aspects of negotiating the meaning of the metaphor (both student↔student and student↔ teacher discussions) to be pedagogically helpful.

In this paper, we give three examples of tangible metaphors that we have chosen from different physics subdisciplines in order to demonstrate their universality. We have also chosen examples that vary in the natural intuitiveness of their use and we have chosen some examples where students use their bodies exclusively in the metaphor and others where the metaphor is primarily carried by external manipulatives which the students can feel. In the conclusion, we discuss further the similarities and differences of these examples.

CHARGE & CURRENT DENSITIES

During the charge densities and current densities activities,[7] students use their own bodies to represent point charges and coordinate with several students to model linear, surface, and volume charge or current densities. The instructor prompts "Make a linear charge (or current) density", and "Does the line have to be straight?" Other discussions of charge/current densities then follow.

The primary goals of the activities (of their many goals) are for students to understand the geometry of linear, surface, and volume charge or current densities (i.e. linear densities exist along curves that are not necessarily straight lines) and for students to build conceptual understanding of densities by discussing idealized measurement processes of counting charges over an amount of space or that pass a "gate" in a period of time. These activities draw on students' familiarity with uniform volume mass densities to think about the less familiar cases of non-uniform densities, linear or surface densities, or densities of discrete particles.

FIGURE 1. Discussion of non-uniform linear charge density. The instructor draws attention to the different spacings between the students with the meter stick.

FIGURE 2. Students act out a constant linear current density by walking in a line that curves.

Material features of this tangible metaphor are the students' bodies (which are discrete entities), the space of the room (over which the students distribute themselves), and a meter stick that identifies a chunk of length, in the case of charge density, or a gate for charges to pass, in the case of current density. These material features of the tangible metaphor align well with the main instructional goals. Each student can choose a location in the room so that each considers the geometry of a particular type of density. Similarly, the spacing between the students, the mobility of the students, and the location and length of the meter stick facilitates discussions about how one conceptually measures various charge or current densities when considering discrete charges.

SPIN ½ STATES

We have found many students were struggling to understand the two complex-component vectors that represent the quantum state. The two components of the state are analogous to the two-components of arrow vectors, but each component, separately, is a complex number. The fact that the overall phase is irrelevant to the physics creates an added complication. Therefore, we have introduced kinesthetic activities in which pairs of students represent the two complex numbers of the spin ½ state with their left arms. Each arm can rotate in a plane that represents the complex plane. A straight arm horizontal to the ground in front of the person represents the real axis and a vertical arm represents the pure imaginary axis, Fig. 3.

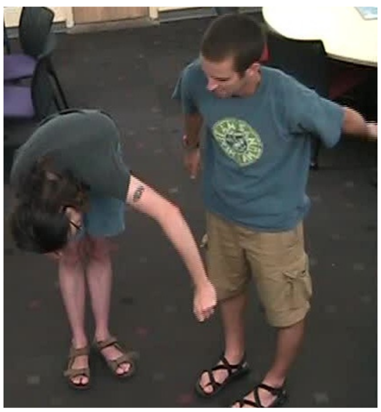

FIGURE 3. A pair of students negotiate using their arms to represent a spin ½ state. Here a student clearly does not realize that his shoulder should represent the fixed origin around which his arm, representing the complex number should rotate.

Initial prompts include "Represent the complex number i" and "Represent the pair of complex numbers 1 and $(1+i)/\sqrt{2}$". The material features of this metaphor are the students' bodies and three-dimensional space. The main goal is for students to understand geometrically the two phases (relative and overall) of the two components of the state. These material features support comparison of components of states in that they treat these components as being separate entities – one for each student – that can vary independently. The students visualize constant relative phases as a constant angle between the two students' arms, and relative phases that change with time as the angle between the two arms changes with time.

Students get clues by looking around to see what others are doing. Instructors also can see which students are understanding and which are not. For example, students who are moving their whole body (see Fig. 4) rather than just their arm do not understand the role of their shoulder as the origin.

Recently, Close *et al.* have developed external representations that they call nested phasor diagrams. [12] As with tangible metaphors, these representations are intended to mediate the conceptual blend between the physics and the mathematics. In their case, however, the representations are kept deliberately in the realm of two-dimensional diagrams and the increasing complexity of the physics is carried out by layering the representations.

PARTIAL DERIVATIVES

In another sequence of activities, students play with a "Partial Derivative Machine" (PDM), see Fig. 5. [7] The PDM consists of an elastic system (initially hidden in a black box) connected to strings, which has two "widths." There are also two pulleys with weights that can be used to adjust the tension in the two directions, giving a total of four tangible degrees of freedom: two forces and two widths, of which only two are independent. There is also a fifth intangible quantity present in this system: the potential energy of the system.

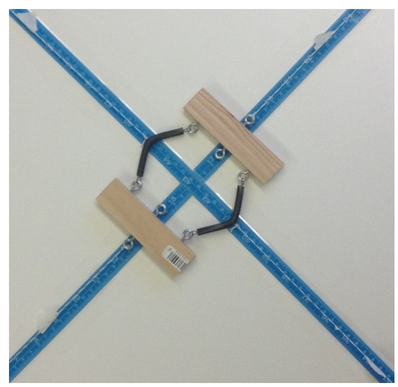

FIGURE 4. The partial derivative machine.

The PDM serves as a tangible metaphor in two senses. First, it is a mechanical analogue for a thermodynamic system, in which two spatial coordinates and two forces play the roles of entropy, volume, temperature, and pressure, while the intangible potential energy plays the role of internal energy in thermodynamics. At the same time, it is a tangible metaphor for the mathematical concept of a partial derivative, with derivatives relating to specific manipulations of the machine. This allows us to teach mathematical concepts within a physical context, allowing students to connect the math with their physical intuition.

When students are initially taught about partial derivatives, they are usually told that it is a derivative in which all the other independent variables are held fixed. They *hear* that all other variables are held fixed, and this usually works just fine until they reach thermodynamics. In thermodynamics, we expect students to explicitly write down which variables are held fixed. We find that students consistently and firmly believe that either "all other variables are fixed," or "it doesn't matter which variable is held fixed." These beliefs interfere with students' abilities to understand and use partial derivative manipulations taught in upper-division thermodynamics. Using the PDM students can measure both the "isowidth stretchability" and the "isoforce stretchability," and observe that for some systems they are quite different.

The metaphor between the mechanical system and thermodynamics is deep at a physical level. The First Law of Thermodynamics is mirrored by the relationship of work to potential energy

$$dU = F_x dx + F_y dy$$

corresponding to the First Law, both mathematically and in its physical meaning. Students take measurements of their system and integrate to find differences in potential energy. Finding an intangible energy from measurements of tangible forces in multiple dimensions leads students to discussions of choices of path (e.g. should they fix F_y while adjusting x or fix y?) and how these choices influence work integrals.

DISCUSSION

These metaphors are designed so that the mapping between the systems and the material features of the objects of the metaphor support the primary learning goals well. However, we have found that productive classroom discussions also arise when the material features of the metaphor do not map particularly well to the physical/mathematical system. For example, two explicit idealizations are highlighted in the process of mapping charge/current density phenomena onto a representation consisting of people. One idealization addresses the question of whether or not all the "charges" are "the same" when the students' bodies come in a variety of shapes and sizes. Another idealization addresses whether the students' manifestly discrete three dimensional bodies make a good model for zero-dimensional point particles or smooth distributions.

These tangible metaphors vary in how natural the metaphor is for students, the level of idealization involved in the metaphor, and how rich the metaphor is. The charge/current densities metaphor is quite natural and is established quickly for the students, while the partial derivatives machine metaphor takes longer. However, the charge/current densities metaphor requires more idealization than the partial derivatives machine and is correspondingly less rich. Because the partial derivatives machine metaphor models partial derivatives well, many relationships between partial derivatives can be explored.

Some research questions that we will pursue in the future include: How effective are tangible metaphors in achieving their primary (and other) goals? How do the material features of these metaphors support or constrain student learning? To what extent do the various properties of the tangible metaphors (such as how natural the metaphor is, level of idealization, and richness) impact their instructional effectiveness? How can tangible metaphors be optimally sequenced with other instructional strategies? How do students use these tangible metaphors to mediate between the physical/geometric properties of the object and the algebraic representations.

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