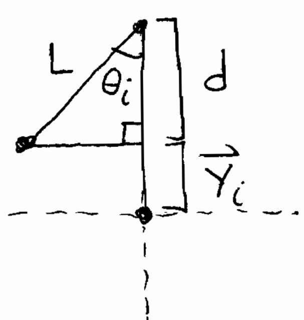
PH 211 – Chapter 10 Homework Solutions

7) $m_o = 4m_H$ $\quad K_o = K_H$ $\quad v_H/v_o = ?$

$$\frac{K_H}{K_o} = \frac{\frac{1}{2}m_H v_H^2}{\frac{1}{2}m_o v_o^2} = \frac{m_H v_H^2}{m_o v_o^2} = 1 \qquad \frac{v_H}{v_o} = \sqrt{\frac{m_o}{m_H}} = \sqrt{\frac{4m_H}{m_H}} = 2.00$$

11) $m = 500\,g = 0.500\,kg$ $\quad L = 0.750\,m$ $\quad \theta_i = 30.0^\circ$ $\quad v_i = 0$

(a)



$\vec{Y}_f = 0 \qquad \cos\theta_i = \frac{adj}{hyp} = \frac{d}{L}$

$d = L\cos\theta_i \qquad d + \vec{Y}_i = L$

$\vec{Y}_i = L - d = L - L\cos\theta_i$

$\vec{Y}_i = L(1-\cos\theta_i)$

$K_i + U_{gi} = K_f + U_{gf} \qquad 0 + mg\vec{Y}_i = \frac{1}{2}mv_f^2 + 0$

$v_f = \sqrt{2g\vec{Y}_i} = \sqrt{2gL(1-\cos\theta_i)} = \left(2(9.80)(0.750)(1-\sqrt{3}/2)\right)^{1/2}$

$v_f = 1.40\,m/s$ $\qquad$ (b) $\theta_f = \theta_i = 30.0^\circ$ due to symmetry

34) $v_i = 80.0\,m/s$ $\quad \vec{Y}_i = +10.0\,m$ $\quad K_i + U_{gi} = K_f + U_{gf}$

$v_f = ?$ $\quad \vec{Y}_f = 0$ $\quad \frac{1}{2}mv_i^2 + mg\vec{Y}_i = \frac{1}{2}mv_f^2 + 0$

$v_f = \sqrt{v_i^2 + 2g\vec{Y}_i} = \left((80.0)^2 + 2(9.80)(10.0)\right)^{1/2} = 81.2\,m/s$

35) (a) No, because the gravitational potential energy and kinetic energy do not depend on the angle or direction.

(b) $K_i + U_{gi} = K_f + U_{gf} \qquad \frac{1}{2}mv_i^2 + mg\vec{Y}_i = \frac{1}{2}mv_f^2 + mg\vec{Y}_f$

$v_i = \sqrt{v_f^2 + 2g\vec{Y}_f - 2g\vec{Y}_i} = \left((5.00)^2 + 2(9.80)(16.0-2.00)\right)^{1/2}$

$v_i = 17.3\,m/s$

38) With two springs, the elastic potential energy is twice as big as it was with only one spring.

$$\frac{2U_s}{U_s} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_0^2} \qquad \frac{v^2}{v_0^2} = 2 \qquad v = \sqrt{2}\,v_0$$

39) With two springs, each spring is compressed only half the distance.

Two springs: $U_s = 2\left(\frac{1}{2}k\left(\frac{\Delta x}{2}\right)^2\right) = \frac{1}{4}k(\Delta x)^2 = \frac{1}{2}\left(\frac{1}{2}k(\Delta x)^2\right)$

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$$\frac{\frac{1}{2}U_s}{U_s} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_0^2} \qquad \frac{v^2}{v_0^2} = \frac{1}{2} \qquad v = \frac{v_0}{\sqrt{2}}$$

40) (a) $\vec{v}_{iB} = -\sqrt{2gh} = -(2(9.80)(10.0))^{1/2}$ $\qquad \vec{v}_{ie} = 0$

$\vec{v}_{fB} = +\sqrt{2gh} = +(2(9.80)(10.0))^{1/2}$ $\qquad \vec{v}_{fe} = ?$

Conservation of Momentum: $m_B\vec{v}_{iB} + m_e\vec{v}_{ie} = m_B\vec{v}_{fB} + m_e\vec{v}_{fe}$

$$\vec{v}_{fe} = \frac{m_B\vec{v}_{iB} - m_B\vec{v}_{fB}}{m_e} = \frac{m_B(-\sqrt{2gh} - \sqrt{2gh})}{m_e}$$

$$\vec{v}_{fe} = \frac{-2m_B\sqrt{2gh}}{m_e} = \frac{-(2)(0.500)(2(9.80)(10.0))^{1/2}}{(5.98\times10^{24})}$$

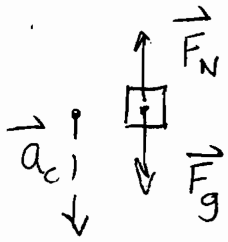
$$\vec{v}_{fe} = -2.34\times10^{-24}\ \text{m/s}$$

(b) $v = \frac{\Delta y}{\Delta t} \qquad \Delta t = \frac{\Delta y}{v} = \frac{(1.00\times10^{-3})}{(2.34\times10^{-24})} = 4.2714\times10^{20}\ \text{s}$

$$\Delta t = 4.2714\times10^{20}\ \text{s}\ \frac{1}{(86400\ \text{s/day})(365.25\ \text{day/yr})}$$

$$\Delta t = 1.35\times10^{13}\ \text{years}$$

53) (a) Find the maximum speed first, which will occur when the object continues to move in a circle and the normal force approaches zero.



$$\Sigma \vec{F}_y = F_N - F_g = -ma_c \qquad 0 - F_g = -ma_c$$

$$F_g = ma_c \qquad mg = m\frac{v^2}{R} \qquad v^2 = gR \quad v = \sqrt{gR}$$

$h_{max} = ?$ $\qquad K_i + U_{gi} = K_f + U_{gf}$

$$0 + mgh = \tfrac{1}{2}mv^2 + mgR \qquad mgh = \tfrac{1}{2}mgR + mgR$$

$$h = \tfrac{1}{2}R + R \qquad h_{max} = \tfrac{3}{2}R$$

(b) $h_{max} = \frac{3}{2}(10.0) = 15.0\,\text{m}$

60) (a) The nitrogen atom is either above or below the xy-plane containing the three hydrogen atoms. It oscillates back and forth about its equilibrium position.

(b) With a total energy greater than the potential energy at $z=0$, the nitrogen atom has enough energy to oscillate about $z=0$, back and forth (or up and down) across the xy-plane.

61) Equilibrium $\quad F_x = \frac{dU}{dx} = 0 \qquad$ min = stable $\qquad$ max = unstable

(a) $\frac{dU}{dx} = 1 + 2\cos 2x = 0 \qquad 2\cos 2x = -1 \qquad \cos 2x = -\frac{1}{2}$

$$2x = \cos^{-1}\left(-\tfrac{1}{2}\right) = \tfrac{2\pi}{3}\,\text{rad},\ \tfrac{4\pi}{3}\,\text{rad} \qquad x = \tfrac{\pi}{3}\,\text{rad},\ \tfrac{2\pi}{3}\,\text{rad}$$

(b) $\frac{d^2U}{dx^2} = -4\sin 2x \qquad \frac{d^2U}{dx^2}\left(x = \frac{\pi}{3}\,\text{rad}\right) = -4\sin\frac{2\pi}{3} = -2\sqrt{3}$

$\frac{d^2U}{dx^2}\left(x = \frac{\pi}{3}\,\text{rad}\right) < 0 \qquad$ max = unstable

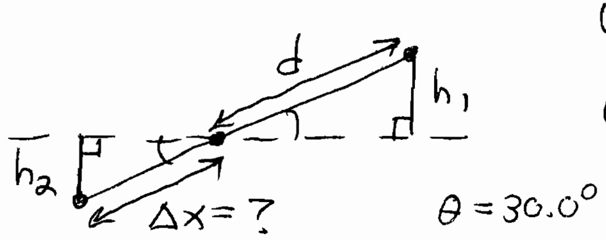
$\frac{d^2U}{dx^2}\left(x = \frac{2\pi}{3}\,\text{rad}\right) = -4\sin\frac{4\pi}{3} = +2\sqrt{3} > 0 \qquad$ min = stable

71) $m = 10.0\,kg \qquad d = 4.00\,m \qquad k = 250\,N/m$

(a) At maximum compression, the speed is zero. The initial speed is also zero.

$U_{gi} = mgh_1 \qquad U_{gf} = -mgh_2$

$U_{si} = 0 \qquad U_{sf} = \frac{1}{2}k(\Delta x)^2$



$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mgh_1 + 0 = 0 - mgh_2 + \frac{1}{2}k(\Delta x)^2$$

$$\sin 30^\circ = \frac{h_2}{\Delta x} \qquad h_2 = \Delta x \sin 30^\circ = \frac{\Delta x}{2}$$

$$\sin 30^\circ = \frac{h_1}{d} \qquad h_1 = d \sin 30^\circ = \frac{d}{2}$$

$$mg\frac{d}{2} = -mg\frac{\Delta x}{2} + \frac{1}{2}k(\Delta x)^2$$

$$k(\Delta x)^2 - mg\,\Delta x - mgd = 0$$

$$\Delta x = \frac{-(-mg) \pm \sqrt{(-mg)^2 - 4k(-mgd)}}{2k} = \frac{mg \pm \sqrt{(mg)^2 + 4kmgd}}{2k}$$

Since $\Delta x > 0$, must use plus sign.

$$\Delta x = \frac{(10.0)(9.80) + \left(((10.0)(9.80))^2 + (4)(250)(10.0)(9.80)(4.00)\right)^{1/2}}{(2)(250)}$$

$$\Delta x = \frac{98 + 633.72}{500} = 1.46\,m$$

(b) $K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$

$$0 + mg\frac{d}{2} + 0 = K_f - mg\frac{\Delta x}{2} + \frac{1}{2}k(\Delta x)^2$$

$$K_f = -\frac{1}{2}k(\Delta x)^2 + mg\frac{\Delta x}{2} + mg\frac{d}{2}$$

Maximum speed equates to maximum kinetic energy.

$$\frac{dK_f}{d\Delta x} = -k\Delta x + \frac{mg}{2} + 0 = 0 \qquad k\Delta x = \frac{mg}{2}$$

$$\Delta x = \frac{mg}{2k} = \frac{(10.0)(9.80)}{(2)(250)} = 0.196\text{ m} = 19.6\text{ cm}$$