

Print your name here: _____ **KEY** _____

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Directions: No books or notes may be used during this exam. Important formulas are provided for you. Graphing calculators and palmtop or laptop computers are not allowed. Always include **three** significant digits in your numerical answers.

Multiple Choice (5 pts. each) You will be graded on your answer only. It is not necessary to show your work for these problems. Circle the letter corresponding to your answer. If needed, then use the back of this page for scratch paper. It will not be read by the grader.

- B** 1. Electrons of maximum kinetic energy K_1 are released when x-rays of wavelength λ_1 are incident on the surface of a metal. If the light source is replaced by another that emits photons at the same rate but of shorter wavelength λ_2 , then how is the resulting maximum kinetic energy K_2 related to K_1 ? (A) $K_2 = K_1$ (**B**) $K_2 > K_1$ (C) $K_2 < K_1$ (D) There is not enough information to determine a unique relationship between K_2 and K_1 .
- A** 2. Which of the following does *not* provide evidence for the wave nature of electrons? (**A**) photoelectric effect (B) tunneling diodes (C) electron diffraction.
- C** 3. The horizontal position of an electron is always greater than zero ($x > 0$) and can be finite or infinite. The total energy of the electron is always greater than its potential energy. Which function could be the solution to the Schrodinger equation for this electron, given that A is the normalization constant and k is the wave number? (A) $A e^{kx}$ (B) $A e^{-kx}$ (**C**) $A \sin kx$ (D) $A \cos kx$ (E) $A \tan kx$ (F) None of these functions could be a solution.
- C** 4. An object of proper length 5.00 m moves with respect to an observer who measures its length to be 3.00 m. What is the speed of the object as measured by the observer? (A) $0.400c$ (B) $0.600c$ (**C**) $0.800c$.
- D** 5. Which of the following sets of quantum numbers (n, l, m_l) is *not* allowed for an electron in a hydrogen atom? (A) (3, 0, 0) (B) (3, 2, -2) (C) (2, 1, 1) (**D**) (2, 2, -1).
- A** 6. What is the de Broglie wavelength of an electron that has 17.0 eV of kinetic energy? (**A**) 0.300 nm (B) 73.0 nm (C) 0.00240 nm (D) 1.60 nm.
- B** 7. Which of the following statements is correct for hydrogen atoms? (A) In the 2s state, the magnitude of the orbital angular momentum is $\sqrt{6}\hbar$. (**B**) In the 3p state, the orbital angular momentum vector can point in three different directions. (C) ($n = 4, l = 2, m_l = -3$) is an allowed set of quantum numbers.
- D** 8. In a Rutherford scattering experiment, what is the approximate ratio of the number of particles scattered at $\theta = 8.00^\circ$ to the number scattered at $\theta = 4.00^\circ$? (A) 1/2 (B) 1/4 (C) 1/8 (**D**) 1/16 (E) 1/32 (F) 1/64.

For the following four problems, show your solutions in a well-organized fashion. Partial credit will be given for proper translations and for identification of the relevant equation(s). **No credit is given for an answer without showing HOW you got the answer.** Use the back of each page if necessary.

9. (40 pts.) **Blue Ray Technology**

While operating for an unknown amount of time, a high-powered laser emits light at a rate of 2.50×10^{18} photons/second. The wavelength of each photon is 480 nm, which is in the blue part of the visible spectrum. What is the average power output of the laser? Give your answer in watts.

Translate

$$\frac{N}{t} = 2.50 \times 10^{18} \text{ photons/s}$$

$$P(\text{ave}) = ?$$

$$\lambda = 480 \text{ nm} = 4.80 \times 10^{-7} \text{ m}$$

$$h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$$

$$c = 2.998 \times 10^8 \text{ m/s}$$

Equate

$$P(\text{ave}) = \frac{E}{t}$$

$$E_{\gamma} = hf$$

$$c = f\lambda$$

Solve

$$P(\text{ave}) = \frac{E}{t} = \frac{N}{t} \frac{E}{N} = \frac{N}{t} hf = \frac{N}{t} h \frac{c}{\lambda}$$

$$P(\text{ave}) = \left(\frac{N}{t}\right) \frac{hc}{\lambda} = \frac{(2.50 \times 10^{18})(6.626 \times 10^{-34})(2.998 \times 10^8)}{(4.80 \times 10^{-7})}$$

$$P(\text{ave}) = 1.03 \times 10^0 = 1.03 \text{ W}$$

10. (40 pts.) Hydrogen Plasma

A hot plasma of hydrogen atoms can be considered to obey Maxwell-Boltzmann statistics. If so, then what is the ratio of the number of atoms in the first excited state to the number of atoms in the ground (lowest energy) state if the gas is at a temperature of 10,000 K? Assume the density of states is constant.

Translate

$$\frac{N_2}{N_1} = ?$$

$$T = 10,000 \text{ K}$$

$$k = 8.617 \times 10^{-5} \text{ eV/K}$$

$$g(E) = \text{constant}$$

$$Z = 1 \text{ for hydrogen}$$

Equate

$$E_n = (-13.6 \text{ eV}) \frac{Z^2}{n^2} \quad n \in \{1, 2, 3, \dots\}$$

$$f_{\text{MB}}(E) = \frac{1}{A e^{E/kT}}$$

Solve

$$\frac{N_2}{N_1} = \frac{p_2(E)}{p_1(E)} = \frac{g_2(E) f_2(E)}{g_1(E) f_1(E)} = \frac{f_2(E)}{f_1(E)}$$

$$\frac{N_2}{N_1} = \frac{A^{-1} e^{-E_2/kT}}{A^{-1} e^{-E_1/kT}} = e^{(E_1 - E_2)/kT} = e^{(-13.6 \text{ eV}) \left(\frac{1}{1} - \frac{1}{4} \right) / kT}$$

$$\frac{N_2}{N_1} = e^{- (13.6 \text{ eV}) \left(\frac{3}{4} \right) / (8.617 \times 10^{-5}) (10,000)} = e^{-11.837}$$

$$\frac{N_2}{N_1} = 7.23 \times 10^{-6} = 7.23 \times 10^{-4} \%$$

11. Virtual Reality

A virtual pion is exchanged between two nucleons in a cesium nucleus. Its probability wave function is

$$\psi(x) = Ae^{-nx/2} \text{ where } x > 0, n = \text{principal quantum number and } A = \text{normalization constant.}$$

Consider the virtual pion to be a free particle so that its potential energy is zero.

- (a) (20 pts.) Determine the normalization constant.
(b) (20 pts.) Determine the quantized energy levels.

Leave your expressions in simplest form for full credit. Use the back of this piece of paper if necessary.

$$(a) \int_0^{\infty} |\psi(x)|^2 dx = 1 \quad A = ?$$
$$\int_0^{\infty} A^2 e^{-nx} dx = A^2 \left[-\frac{e^{-nx}}{n} \right]_0^{\infty} = A^2 \left[0 - \left(-\frac{1}{n} \right) \right] = 1$$
$$A^2 \frac{1}{n} = 1 \quad A = \sqrt{n}$$

$$(b) U=0 \quad -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi(x) \quad \frac{d\psi}{dx} = -\frac{n}{2}\psi \quad \frac{d^2\psi}{dx^2} = \frac{n^2}{4}\psi$$

$$-\frac{\hbar^2}{2m} \frac{n^2}{4} \psi = E\psi \quad E = -\frac{\hbar^2 n^2}{8m}$$

12. (40 pts.) Particle Decay

In the rest frame, a positive pion at rest decays into a positive muon and a muon neutrino: $\pi^+ \rightarrow \mu^+ + \nu_\mu$. Calculate the kinetic energies of each of the two decay products in the rest frame. The rest masses of the particles are as follows: pion $140 \text{ MeV}/c^2$, muon $105.7 \text{ MeV}/c^2$, and neutrino $0 \text{ MeV}/c^2$.

Translate

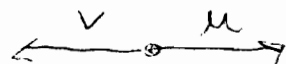
$$\vec{p}_\pi = 0 \quad K_\mu = ? \quad K_\nu = ? \quad m_\pi = 140 \text{ MeV}/c^2$$

$$m_\mu = 105.7 \text{ MeV}/c^2 \quad m_\nu = 0$$

Equate

Conservation of Energy $E_\pi = E_\mu + E_\nu$

Conservation of Momentum $\vec{p}_\pi = \vec{p}_\mu + \vec{p}_\nu$



$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

Solve

Energy: $m_\pi c^2 = \sqrt{(p_\mu c)^2 + (m_\mu c^2)^2} + p_\nu c$

Momentum: $0 = p_\mu - p_\nu \quad p_\mu = p_\nu = p$

$$m_\pi c^2 = \sqrt{(pc)^2 + (m_\mu c^2)^2} + pc \quad \sqrt{(pc)^2 + (m_\mu c^2)^2} = m_\pi c^2 - pc$$

$$(pc)^2 + (m_\mu c^2)^2 = (m_\pi c^2)^2 - 2m_\pi c^2 pc + (pc)^2$$

$$-2m_\pi c^2 pc = (m_\mu c^2)^2 - (m_\pi c^2)^2 \quad pc = \frac{(m_\pi c^2)^2 - (m_\mu c^2)^2}{2m_\pi c^2}$$

$$pc = \frac{(140)^2 - (105.7)^2}{2(140)} = 30.09825 \text{ MeV}$$

$$K_\nu = E_\nu - (m_\nu c^2) = pc - 0 = pc$$

$$K_\nu = 30.1 \text{ MeV}$$

$$K_\mu = E_\mu - (m_\mu c^2) = \sqrt{(pc)^2 + (m_\mu c^2)^2} - (m_\mu c^2)$$

$$K_\mu = \sqrt{(30.09825)^2 + (105.7)^2} - 105.7$$

$$K_\mu = 4.20 \text{ MeV}$$