PH 314 Homework Set #1 Solutions

1) $K = 1.60 \times 10^{-19}\ \text{J}$ $\quad K = \dfrac{p^2}{2m}$ $\quad p^2 = 2mK$ $\quad p = \sqrt{2mK}$

(a) $m_e = 9.11 \times 10^{-31}\ \text{kg}$ $\quad p = ((2)(9.11\times10^{-31})(1.60\times10^{-19}))^{1/2} = 5.40\times10^{-25}\ \text{kg}\cdot\text{m/s}$

(b) $m_p = 1.67 \times 10^{-27}\ \text{kg}$ $\quad p = ((2)(1.67\times10^{-27})(1.60\times10^{-19}))^{1/2} = 2.31\times10^{-23}\ \text{kg}\cdot\text{m/s}$

(c) Since $m_n \cong m_p$, $p = 2.31\times10^{-23}\ \text{kg}\cdot\text{m/s}$

2) (a) Conservation of Energy $(U_e + K)_{initial} = (U_e + K)_{final}$

$$(QV_0 + 0) = (QV + \tfrac{1}{2}mv^2) \qquad \tfrac{1}{2}mv^2 = |Q\Delta V|$$

$$v = \sqrt{\frac{2|Q\Delta V|}{m}} = \left(\frac{(2)(1.60\times10^{-19})(1.00)}{(9.11\times10^{-31})}\right)^{1/2} = 5.93\times10^{5}\ \text{m/s}$$

(b) $$v = \left(\frac{(2)(1.60\times10^{-19})(1.00)}{(1.67\times10^{-27})}\right)^{1/2} = 1.38\times10^{4}\ \text{m/s}$$

3) (a) The clock is running slow, i.e. the other clock displays an elapsed time of more than one minute (sixty seconds).

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ $\qquad \Delta t \geq \Delta t_0$ $\qquad \Delta t_0 = 60$ seconds $\qquad \Delta t = ?$

$$u = \frac{\Delta y}{\Delta t} = \frac{y - y_0}{\Delta t} = \frac{y}{\Delta t} \qquad \Delta t = \frac{\Delta t_0}{\sqrt{1-(y/\Delta t)^2/c^2}}$$

$$(\Delta t)^2 = \frac{(\Delta t_0)^2}{1-(y/c\Delta t)^2} \qquad (\Delta t)^2(1-(y/c\Delta t)^2) = (\Delta t_0)^2$$

$$(\Delta t)^2 - (y/c)^2 = (\Delta t_0^2) \qquad \Delta t = \sqrt{(\Delta t_0)^2 + (y/c)^2}$$

$$\Delta t = ((60)^2 + (2.50\times10^{10}/3.00\times10^{8})^2)^{1/2} = 103\ \text{seconds}$$

4) (a) $v_x' = v_x - u \qquad u = v_x - v_x' = v\cos\theta - (-v\cos\theta) = 2v\cos\theta$

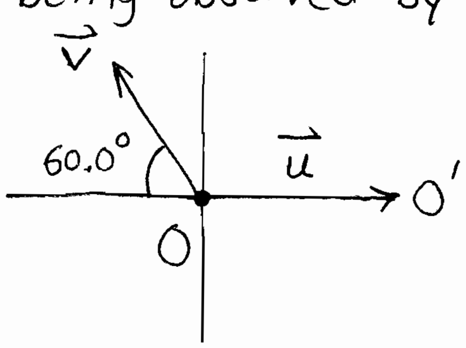
$u = 2(0.800c)(\cos 30.0^\circ) = 1.39c$

No, this speed would not be possible since $u > c$.

(b) By symmetry, $u = 1.39c$. This speed would not be possible for the same reason.

(c) At first glance, it would seem that we can ignore the upward motion just as we did with parts (a) and (b). However, relativistically there will be relative motion along that direction. As the two spaceships move, the horizontal distance between them increases, which increases the amount of time it takes for light signals to go from one spaceship to the other.

In order to determine the relative speed between the two spaceships, we need to use the Lorentz velocity transformations on page 37 of Krane, which assumes that the relative velocity between O and O' is to the right along the positive x-axis. So, we must rotate the axis such that one of the spaceships moves along the positive x-axis from the point of view of observer O on the Earth. Let the Enterprise move along the positive x-axis and let observer O' move with the Enterprise. The Patah is the object being observed by O and O'.



$\vec{u} = (u_x, u_y, u_z) = (0.800c, 0, 0)$

$\vec{v} = (v_x, v_y, v_z) =$

$((-0.800c)\cos\theta, (0.800c)\sin\theta, 0)$

$\theta = 60.0^\circ$

$\vec{v}' = ?$

$\vec{v}'$ is the velocity of the Patah as measured by Observer O' who moves with the Enterprise.

$$v_x' = \frac{v_x - u}{1 - v_x u/c^2} = \frac{((-0.800c)(\cos 60.0°) - (0.800c)}{1 - (-0.800c)(\cos 60.0°)(0.800c)/c^2} = -0.9091c$$

$$v_y' = \frac{v_y\sqrt{1-u^2/c^2}}{1 - v_x u/c^2} = \frac{(0.800c)(\sin 60.0°)\sqrt{1-(0.800c)^2/c^2}}{1-(-0.800c)(\cos 60.0°)(0.800c)/c^2} = +0.3149c$$

$$v_z' = \frac{v_z\sqrt{1-u^2/c^2}}{1 - v_x u/c^2} = 0 \quad \text{since } v_z = 0$$

v' = speed of Patah as measured by O' moving with the Enterprise.

$$v' = \sqrt{(v_x')^2 + (v_y')^2 + (v_z')^2}$$

$$v' = ((-0.9091c)^2 + (+0.3149c)^2 + (0)^2)^{1/2} = 0.962c$$

(d) By symmetry, $v' = 0.962c$.