PH 314 Homework Set #4 Solutions

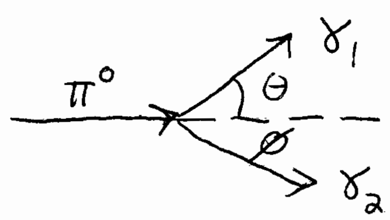
1) The easiest way to prove that it is not possible is to switch to a reference frame in which the electron is initially stationary. This reference frame is called the rest frame. If the electron is initially stationary, then the total momentum initially is zero and the total energy initially is equal to the rest energy of the electron.

After the electron emits the photon, in the rest frame, the electron and photon would move in opposite directions, and momentum would be conserved. However, total energy would not be conserved since the kinetic energy of the electron and the energy of the photon are both greater than zero. In other words, the total energy increases in the rest frame, which violates conservation of energy.

This can also be proved in the original reference frame. However, there would be much algebra needed to prove that the total energy increases.

2) $v_\pi = 0.800c$

$m_\pi = 135\ \text{MeV}/c^2$



$E_{\gamma_1} = 2E_{\gamma_2}$

(a) $\theta + \phi = ?$

Conservation of Momentum: x: $p_\pi = p_{\gamma_1}\cos\theta + p_{\gamma_2}\cos\phi$

Conservation of Momentum: y: $0 = p_{\gamma_1}\sin\theta - p_{\gamma_2}\sin\phi$

Conservation of Energy: $\dfrac{mc^2}{\sqrt{1-(v_\pi/c)^2}} = p_{\gamma_1}c + p_{\gamma_2}c$

Solve these equations simultaneously.

From y-momentum: $p_{\gamma_1}\sin\theta = p_{\gamma_2}\sin\phi \qquad \frac{E_{\gamma_1}}{c}\sin\theta = \frac{E_{\gamma_2}}{c}\sin\phi$

$$2E_{\gamma_2}\sin\theta = E_{\gamma_2}\sin\phi \qquad \boxed{2\sin\theta = \sin\phi}$$

From x-momentum: $p_\pi = \frac{E_{\gamma_1}}{c}\cos\theta + \frac{E_{\gamma_2}}{c}\cos\phi$

$$p_\pi c = 2E_{\gamma_2}\cos\theta + E_{\gamma_2}\cos\phi \qquad p_\pi c = E_{\gamma_2}(2\cos\theta + \cos\phi)$$

$$p_\pi = \frac{mv_\pi}{\sqrt{1-(v_\pi/c)^2}} = \frac{m(0.800c)}{\sqrt{1-(0.800c/c)^2}} = \frac{4}{3}mc \qquad p_\pi c = \frac{4}{3}mc^2$$

$$\boxed{\frac{4}{3}mc^2 = E_{\gamma_2}(2\cos\theta + \cos\phi)}$$

From energy: $\frac{mc^2}{\sqrt{1-(v_\pi/c)^2}} = \frac{mc^2}{\sqrt{1-(0.800c/c)^2}} = \frac{5}{3}mc^2 = E_{\gamma_1} + E_{\gamma_2}$

$$\frac{5}{3}mc^2 = 2E_{\gamma_2} + E_{\gamma_2} = 3E_{\gamma_2} \qquad \boxed{E_{\gamma_2} = \frac{5}{9}mc^2}$$

Solve the three equations simultaneously.

$$\frac{4}{3}mc^2 = \frac{5}{9}mc^2(2\cos\theta + \cos\phi) \qquad \frac{12}{5} = 2\cos\theta + \cos\phi$$

$$\cos\phi = \frac{12}{5} - 2\cos\theta \qquad \cos^2\phi = \left(\frac{12}{5} - 2\cos\theta\right)^2$$

$\sin^2\phi = 4\sin^2\theta$ (from y-momentum)

$$\sin^2\phi + \cos^2\phi = 1 = 4\sin^2\theta + \left(\frac{12}{5} - 2\cos\theta\right)^2$$

$$4\sin^2\theta + 4\cos^2\theta - \frac{48}{5}\cos\theta + \frac{144}{25} = 1$$

$$4(\sin^2\theta + \cos^2\theta) - \frac{48}{5}\cos\theta + \frac{144}{25} = 1 \qquad 4 - \frac{48}{5}\cos\theta + \frac{144}{25} = 1$$

$$\frac{48}{5}\cos\theta = 3 + \frac{144}{25} \qquad \frac{240}{25}\cos\theta = \frac{75+144}{25} = \frac{219}{25}$$

$$\cos\theta = \frac{219}{240} \qquad \theta = \cos^{-1}\left(\frac{219}{240}\right) = 24.1^\circ \qquad \boxed{\theta = 24.1^\circ}$$

$$\sin\phi = 2\sin\theta = 2\left(\sqrt{1-\cos^2\theta}\right) = 2\left(\sqrt{1-(219/240)^2}\right)$$

$$\phi = \sin^{-1}\left(2\sqrt{1-(219/240)^2}\right) = 54.9^\circ$$

$$\theta + \phi = 24.1^\circ + 54.9^\circ = 79.0^\circ$$

(b) $E_{\gamma_2} = \frac{5}{9}mc^2 \qquad hf_2 = \frac{5}{9}mc^2 \qquad h\frac{c}{\lambda_2} = \frac{5}{9}mc^2$

$$\lambda_2 = \frac{hc}{\frac{5}{9}mc^2} = \frac{9(hc)}{5(mc^2)} = \frac{9(1240\ \text{eV}\cdot\text{nm})}{5(135\ \text{MeV})}$$

$$\lambda_2 = 16.5\ \text{fm} = 1.65\times10^{-14}\ \text{m}$$

$E_{\gamma_1} = 2E_{\gamma_2}$ Photon has twice the energy. Wavelength is inversely proportional to energy

$$\lambda_1 = \frac{1}{2}\lambda_2 = 8.27\ \text{fm} = 8.27\times10^{-15}\ \text{m}$$

3) $\lambda = \frac{h}{p}$ We can use $p = mv$ since $v \ll c$.

$$\lambda = \frac{(6.626\times10^{-34})}{(7.25)(12.5)} = 7.31\times10^{-36}\ \text{m}$$

4) $V = \frac{4}{3}\pi R^3 \qquad \rho = 745\ \text{kg/m}^3 \qquad v = ? \qquad \lambda = R$

Assume $v \ll c$. $\quad \lambda = \frac{h}{p} = \frac{h}{mv} \qquad m = 142\text{g} = 0.142\text{kg}$

$$m = \rho V = \rho\frac{4}{3}\pi R^3 \qquad R = \left(\frac{3m}{4\pi\rho}\right)^{1/3}$$

$$\frac{h}{mv} = \left(\frac{3m}{4\pi\rho}\right)^{1/3} \qquad v = \frac{h}{m}\left(\frac{4\pi\rho}{3m}\right)^{1/3} = h\left(\frac{4\pi\rho}{3}\right)^{1/3}(m)^{-4/3}$$

$$v = (6.626\times10^{-34})\left(\frac{4\pi(745)}{3}\right)^{1/3}(0.142)^{-4/3} = 1.31\times10^{-31}\ \text{m/s}$$

5) $\lambda = 1.25\,nm = 1.25 \times 10^{-9}\,m$

(a) $k = \text{wave number} = \frac{2\pi}{\lambda} = \frac{2\pi}{1.25 \times 10^{-9}} = 5.03 \times 10^{9}\ rad/m$

(b) $\lambda = \frac{h}{p} \qquad p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{1.25 \times 10^{-9}} = 5.30 \times 10^{-25}\ kg \cdot m/s$

(c) Do we need to use relativity? Let's see.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$$

$$K = \frac{(5.30 \times 10^{-25})^2}{(2)(9.11 \times 10^{-31})} = 1.54 \times 10^{-19}\ J \cong 1eV$$

$$mc^2 = 0.511\ MeV = 511{,}000\ eV$$

$K << mc^2$ so classical model is OK. $K = 1.54 \times 10^{-19}\ J$

6) The uncertainty in the energy is $\Delta E = 120\ MeV = 1.20 \times 10^{8}\ eV$
The lifetime of the Δ particle is the uncertainty in the time Δt.

From the Heisenberg Uncertainty Principle, $\Delta E \Delta t \sim \hbar$

$$\Delta t \sim \frac{\hbar}{\Delta E} = \frac{(6.58 \times 10^{-16}\ eV \cdot s)}{(1.20 \times 10^{8}\ eV)} = 5.48 \times 10^{-24}\ s$$