PH 314 Midterm Exam 05/07/2008

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Directions: No books or notes may be used during this exam. Important formulas are provided for you. Graphing calculators and palmtop or laptop computers are not allowed. Always include **three** significant digits in your numerical answers.

$c = 2.998 \times 10^8$ m/s $\quad e = 1.602 \times 10^{-19}$ C $\quad k = 1.381 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K

$h = 6.626 \times 10^{-34}$ J·s $= 4.136 \times 10^{-15}$ eV·s $\quad hc = 1240$ eV·nm $= 1240$ MeV·fm $\quad \hbar = h/2\pi$

1 eV $= 1.602 \times 10^{-19}$ J $\quad$ 1 u $= 931.50$ MeV/$c^2$ $= 1.6605 \times 10^{-27}$ kg

$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/$c^2$ $\quad m_p = 938.3$ MeV/$c^2$ $\quad m_n = 939.6$ MeV/$c^2$

$h/m_e c = 0.002426$ nm $\quad \sigma = 5.67 \times 10^{-8}$ W/m$^2$·K$^4$ $\quad$ 1 nm $= 10^{-9}$ m $\quad$ 1 fm $= 10^{-15}$ m

$N_A = 6.02 \times 10^{23}$ $\quad R = 8.314$ J/mole·K $\quad e^2/4\pi\varepsilon_0 = 1.44\ \text{eV}\cdot\text{nm}$

$a_0 = 4\pi\varepsilon_0\hbar^2/me^2 = 0.0529\,\text{nm} \quad R_\infty = me^4/64\pi^3\varepsilon_0^2\hbar^3 c = 1.097\times 10^7\ \text{m}^{-1} \quad \mu_B = e\hbar/2m = 9.274\times 10^{-24}\ \text{J/T}$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \qquad L = L_0\sqrt{1-u^2/c^2} \qquad v = \frac{v'+u}{1+v'u/c^2}$$

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} \qquad K = E - E_0 \qquad E = \frac{mc^2}{\sqrt{1-v^2/c^2}} \qquad E_0 = mc^2 \qquad E = \sqrt{(pc)^2 + (mc^2)^2}$$

$$PV = nRT \qquad PV = NkT \qquad v_{\text{rms}} = \sqrt{3P/\rho} = \sqrt{3kT/m} \qquad \tfrac{1}{2}m(v^2)_{\text{av}} = \tfrac{3}{2}kT \qquad v_{\text{av}} = \sqrt{8kT/\pi m}$$

$$n(v) = 4\pi N(m/2\pi kT)^{3/2} v^2 e^{-mv^2/2kT} \qquad N(v_1|v_2) = \int_{v_1}^{v_2} n(v)dv \qquad f(v_1|v_2) = \frac{1}{N}\int_{v_1}^{v_2} n(v)dv$$

$$N(E) = 2N\pi^{-1/2}(kT)^{-3/2}E^{1/2}e^{-E/kT} \qquad E_{\text{int}} = \tfrac{3}{2}nRT \text{ (monatomic gas)} \qquad C_V = \Delta E_{\text{int}}/n\Delta T$$

$$E = h\nu \qquad c = \lambda\nu \qquad K_{\max} = eV_s = h\nu - \phi \qquad p = h/\lambda \qquad I = \sigma T^4 \qquad \lambda_{\max}T = 2.898\times 10^{-3}\,\text{m}\cdot\text{K}$$

$$R(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)} \qquad \lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \qquad \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1-\cos\theta) \qquad \lambda_{\min} = \frac{hc}{eV}$$

$$\Delta x\Delta p_x \ge \hbar/2 \qquad \Delta E\Delta t \ge \hbar/2 \qquad v_{\text{phase}} = \omega/k \qquad v_{\text{group}} = d\omega/dk$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x) \qquad P(x)dx = |\psi|^2 dx \qquad P = \int_{x_1}^{x_2} |\psi(x)|^2 dx \qquad \int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$$

$$[f(x)]_{\text{av}} = \int_{-\infty}^{+\infty} |\psi(x)|^2 f(x)dx \qquad \text{Inf. well: } \psi_n(x) = \sqrt{\tfrac{2}{L}}\sin\frac{n\pi x}{L} \qquad E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2} \quad (n = 1,2,3...)$$

$$\Psi(x,t) = \psi(x)e^{-i\omega t} \qquad e^{i\theta} = \cos\theta + i\sin\theta \qquad \text{Osc.: } \psi_0(x) = Ae^{-ax^2} \quad E_n = (n+\tfrac{1}{2})\hbar\omega_0 \quad (n = 0,1,2,...)$$

***Multiple Choice*** (5 pts. each) You will be graded on your answer only. It is not necessary to show your work for these problems. Circle the letter corresponding to your answer. If needed, then use the back of this page for scratch paper. It will not be read by the grader.

1. Observer A measures clock x not moving at all times. Observer A also measures observer B and clock y moving together at the same constant velocity at all times. According to Observer A, during the first event, clock x displays an elapsed time of 5.00 s and clock y displays an elapsed time of 4.00 s. According to Observer B, during the second event, clock y displays an elapsed time of 10.0 s. According to Observer B, what elapsed time is displayed on clock x during the second event? (A) 6.00 s (B) 8.00 s (C) 9.00 s (D) 11.0 s (E) 12.0 s.

2. Which has the largest de Broglie wavelength? (A) a car moving at 25.0 m/s (B) an electron moving at 0.100*c* (C) a proton moving at 0.100*c* (D) a space probe moving at 20.0 km/s (E) the instructor walking at 2.00 m/s.

3. What is the name of this reaction: $e^- + \gamma \rightarrow e^- + \gamma$, where $\gamma$ stands for photon and $e^-$ stands for electron? (A) photoelectric effect (B) Compton scattering (C) bremsstrahlung (D) pair production (E) pair annihilation.

4. The energy of the ground (lowest energy) state of a quantum simple harmonic oscillator is 0.500 eV. What is the energy of its second excited (third lowest energy) state? (A) 1.50 eV (B) 2.00 eV (C) 2.50 eV (D) 3.00 eV (E) 3.50 eV (F) 4.00 eV (G) 4.50 eV (H) 5.00 eV.

5. The probability distribution function for the speeds of ultracold electrons in a plasma screen television is given as $f(v) = 3(4v - v^2)/32$, where $v$ is the speed of the electrons measured in m/sec and $0 < v < 4.00$ m/sec. What is the average kinetic energy of these electrons? The mass of each electron is $9.11 \times 10^{-31}$ kg. (A) $1.82 \times 10^{-30}$ J (B) $2.19 \times 10^{-30}$ J (C) $4.10 \times 10^{-30}$ J (D) $7.29 \times 10^{-30}$ J.

1. $\dfrac{4.00\,s}{5.00\,s} = \dfrac{\Delta t_{2B}}{10.0\,s}$ $\quad \Delta t_{2B} = 8.00\,s$

2. Electron mass is much less than proton mass and $\lambda = \dfrac{h}{mv}$.

3. Electron and photon both exist before and after they interact.

4. $E_n = (n+\frac{1}{2})\hbar\omega_0 \quad E_0 = \frac{1}{2}\hbar\omega_0 \quad E_2 = \frac{5}{2}\hbar\omega_0$

$E_2 = 5E_0 = 5(0.500\,eV) = 2.50\,eV$

5. $\langle K \rangle = \langle \frac{1}{2}mv^2 \rangle = \frac{1}{2}m_e\langle v^2 \rangle = \frac{1}{2}m_e\int_0^4 v^2 f(v)\,dv$

$= \frac{1}{2}m_e\frac{3}{32}\int_0^4 (4v^3 - v^4)\,dv = \frac{3}{64}m_e\left[v^4 - \frac{1}{5}v^5\right]_0^4 = \frac{3}{64}m_e\frac{256}{5} = \frac{12}{5}m_e$

For the following three problems, show your solutions in a well-organized fashion. Partial credit will be given for proper translations and for identification of the relevant equation(s). **No credit is given for an answer without showing HOW you got the answer.**

6. (25 pts.) **A Certain Point of View**

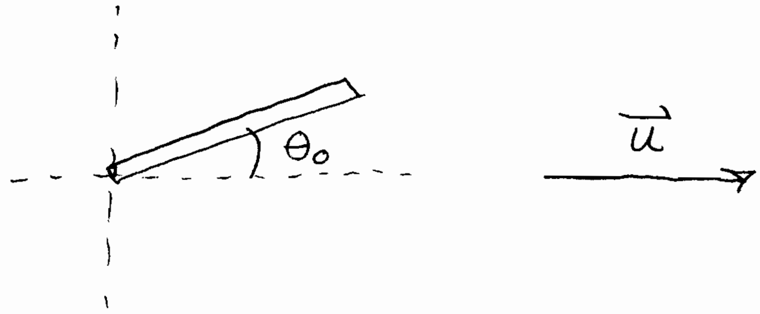
Observer A sees a stationary meterstick (of length 1.00 m) lying in the *xy*-plane. Observer A measures the angle between the meterstick and the *x*-axis as 30.0 degrees. According to Observer A, Observer B moves at 0.800*c* in the positive *x*-direction. According to Observer B, what is the length of the meterstick and what is the angle between the meterstick and the *x*-axis?

Translate

$L_0 = 1.00\text{ m}$

$\theta_0 = 30.0^\circ$

$u = 0.800c$




Equate

$$L = L_0\sqrt{1-u^2/c^2}$$

Solve

The vertical component is unchanged. $\quad Y = Y_0 = L_0 \sin\theta_0$

The horizontal component is contracted.

$$X = X_0\sqrt{1-u^2/c^2} = (L_0\cos\theta_0)\sqrt{1-u^2/c^2} = (L_0\cos\theta_0)\sqrt{1-(0.800)^2}$$

$$X = (L_0\cos\theta_0)\sqrt{1-0.64} = (L_0\cos\theta_0)(0.6)$$

$$L = \sqrt{X^2+Y^2} = \left[((L_0\cos\theta_0)(0.6))^2 + (L_0\sin\theta_0)^2\right]^{1/2}$$

$$L = L_0\left[(0.6\cos 30.0^\circ)^2 + (\sin 30.0^\circ)^2\right]^{1/2} = L_0\left[(0.36)(0.75) + (0.25)\right]^{1/2}$$

$$L = (1.00)(0.7211) = 0.7211\text{ m} = \underline{0.721\text{ m}} = \underline{72.1\text{ cm}}$$

$$\theta = \tan^{-1}\left(\frac{Y}{X}\right) = \tan^{-1}\left(\frac{L_0\sin\theta_0}{0.6L_0\cos\theta_0}\right) = \tan^{-1}\left(\frac{\tan\theta_0}{0.6}\right)$$

$$\theta = \tan^{-1}\left(\frac{\tan 30.0^\circ}{0.6}\right) = 43.89^\circ = \underline{43.9^\circ}$$

7. (25 pts.) **Particle Confinement**

A particle is confined to the region $0 \le x \le L$, where $L$ is finite. Its potential energy is given by the following piecewise function.

| | |
|---|---|
| $U = 0$ | $0 \le x \le L$, |
| $U = \infty$ | otherwise. |

The particle is in its lowest energy state ($n$ = 1). Determine the probability that the particle is located in the region $3L/4 \le x \le L$.

Translate

$$P(3L/4 \le x \le L) = ?$$

Equate

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad P = \int_{x_1}^{x_2} |\psi(x)|^2 \, dx$$

Solve

$$n=1 \quad \psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \qquad |\psi(x)|^2 = \frac{2}{L} \sin^2\left(\frac{\pi x}{L}\right)$$

$$P = \int_{3L/4}^{L} \frac{2}{L} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L} \int_{3L/4}^{L} \left( \frac{1 - \cos\left[2\left(\frac{\pi x}{L}\right)\right]}{2} \right) dx$$

$$P = \frac{1}{L}\left[ x - \frac{L}{2\pi} \sin\left(\frac{2\pi x}{L}\right)\right]_{3L/4}^{L} = \frac{1}{L}\left[ (L - 3L/4) + \left(0 - \frac{L}{2\pi}\right)\right]$$

$$P = \frac{1}{L}\left[\frac{L}{4} - \frac{L}{2\pi}\right] = \frac{1}{4} - \frac{1}{2\pi} = \frac{\pi - 2}{4\pi} = 0.09084$$

$$P = \underline{0.0908} = \underline{9.08\%}$$

8. (25 pts.) **Radioactive Beta Decay**

In radioactive beta decay, a neutron decays into a proton, an electron and an anti-electron neutrino. The reaction written symbolically is $n \rightarrow p^+ + e^- + \nu_e$. Before the decay, the neutron is stationary. After the decay, the proton is stationary. The anti-electron neutrino has a very small mass. For the purposes of this problem, treat the anti-electron neutrino as if it had no mass, i.e. treat it the same way you would treat a (massless) photon that moves at the speed of light. Determine the speed of the electron. Give your answer in terms of the speed of light. (Note: The masses of the neutron, proton and electron can be found on the first page of this exam.)

Translate

$m_e = 0.511\ \text{MeV}/c^2 \qquad m_p = 938.3\ \text{MeV}/c^2 \qquad m_n = 939.6\ \text{MeV}/c^2$

$p_n = 0 \qquad p_p = 0 \qquad m_\nu = 0 \qquad v_e = ?$

Equate

$$E = \sqrt{(pc)^2 + (mc^2)^2} \qquad p = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

Solve

Conservation of Energy: $E_n = E_p + E_e + E_\nu$

$E_n = m_n c^2 \qquad E_p = m_p c^2 \qquad E_e = \sqrt{(p_e c)^2 + (m_e c^2)^2} \qquad E_\nu = p_\nu c$

Conservation of Momentum: $\vec{p}_n = \vec{p}_p + \vec{p}_e + \vec{p}_\nu$

$\vec{p}_n = 0 \qquad \vec{p}_p = 0 \qquad 0 = 0 + \vec{p}_e + \vec{p}_\nu \qquad \vec{p}_e = -\vec{p}_\nu \qquad p_e = p_\nu$

Substitute:

$$m_n c^2 = m_p c^2 + \sqrt{(p_e c)^2 + (m_e c^2)^2} + p_e c$$

$$\sqrt{(p_e c)^2 + (m_e c^2)^2} = (m_n c^2 - m_p c^2) - p_e c \quad \text{(square both sides)}$$

$$(p_e c)^2 + (m_e c^2)^2 = (m_n c^2 - m_p c^2)^2 - 2 p_e c (m_n c^2 - m_p c^2) + (p_e c)^2$$

$$2 p_e c (m_n c^2 - m_p c^2) = (m_n c^2 - m_p c^2)^2 - (m_e c^2)^2$$

$$p_e = \frac{(m_n c^2 - m_p c^2)^2 - (m_e c^2)^2}{2c(m_n c^2 - m_p c^2)} \qquad p_e = \frac{m_e v_e}{\sqrt{1 - v_e^2/c^2}}$$

$$\frac{(m_n c^2 - m_p c^2)^2 - (m_e c^2)^2}{2c(m_n c^2 - m_p c^2)} = \frac{m_e v_e}{\sqrt{1 - v_e^2/c^2}}$$

$$\frac{(939.6 - 938.3)^2 - (0.511)^2}{2c(939.6 - 938.3)} = 0.549568 \ \frac{\text{MeV}}{c}$$

$$\left(0.549568 \ \frac{\text{MeV}}{c}\right)^2 = \left(\frac{m_e v_e}{\sqrt{1 - v_e^2/c^2}}\right)^2 = \frac{m_e^2 v_e^2}{1 - v_e^2/c^2}$$

$$\frac{(\text{MeV})^2}{c^2}(0.3020259)(1 - v_e^2/c^2) = m_e^2 v_e^2$$

$$\frac{(\text{MeV})^2}{c^2}(0.3020259)(c^2 - v_e^2) = (m_e^2 c^2 v_e^2)$$

$$(\underbrace{0.3020259 (\text{MeV})^2}_{A})(c^2 - v_e^2) = (m_e^2 c^4 v_e^2) = (m_e c^2)^2 v_e^2$$

$$A(c^2 - v_e^2) = (m_e c^2)^2 v_e^2 \qquad A c^2 - A v_e^2 = (m_e c^2)^2 v_e^2$$

$$A v_e^2 + (m_e c^2)^2 v_e^2 = A c^2 \qquad v_e^2 (A + (m_e c^2)^2) = A c^2$$

$$v_e^2 = \frac{A c^2}{A + (m_e c^2)^2} \qquad v_e = \sqrt{\frac{A}{A + (m_e c^2)^2}} \ c$$

$$v_e = \sqrt{\left(\frac{0.3020259}{(0.3020259 + (0.511)^2)}\right)} \ c$$

$$= 0.7323c = \underline{0.732c}$$