Sample Problem Formats

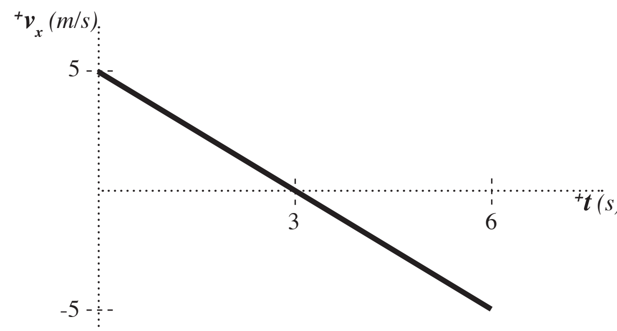
Use the following examples as a guide to various types of solutions to prep and exam problems.

A Sample T/F/N Item

*Here is how an item like this might appear in a Prep set or on an exam. You are being asked to evaluate (**and justify**) each statement as conclusively True (T), conclusively False (F), or with Not enough information (N) to conclude one way or the other.*

1. An object's motion is described by this velocity-time graph. Evaluate (T/F/N) each statement below if it is applied to the entire 6 s of motion shown.

(A) The object's acceleration (vector), $\boldsymbol{a}$, was constant.
(B) The object's displacement (vector), $\Delta\boldsymbol{x}$, was +15 m.
(C) The object was moving under the influence of gravity only.



Below is one example of a completely correct set of answers (and of course, the answers could be any combination of T, F and/or N—not necessarily one of each). Note: <u>The wording of the explanations will be different</u> for each person (and for any answer of F or N, citing just one counter-example or exception is sufficient). <u>So these could be more succinct (so clear phrases rather than complete sentences are fine) and still considered acceptable</u>. The versions given below include <u>more than enough</u> discussion in order to show you the full range of what <u>could</u> be offered to explain this problem's answers.

1	(A) The acceleration vector value is the slope of the velocity-time graph, and the slope of the graph given here is constant.	A	**T**
	(B) The displacement vector value is given by the area under the velocity-time graph, <u>which is about **zero** here, judging from the data given by the graph</u>; the object's direction of motion *reverses* at the 3 s mark, halfway through the time interval.	B	**F**
	(C) The "*x*-direction" is NOT always horizontal; we can select any axis names/directions we wish. So it is <u>possible</u> that this is indeed a projectile, initially moving vertically upward (although judging from the graph's slope, the magnitude of the gravitational acceleration more nearly matches that on moon's surface than here on earth). But this <u>could also</u> be, say, a rocket sled that is steadily slowing, then reversing, its horizontal motion.	C	**N**

*Note: For an item with <u>three</u> statements to evaluate (such as the above), often you'll be provided with an answer area as shown here. For other T/F/N items—possibly containing more than three statements—you'll just be given blank space, but **you should evaluate each statement in the same manner as shown here**—with both a clearly marked <u>answer</u> (T/F/N) **and an accompanying explanation**.*

*In any T/F/N/ item, each statement (so A, B, and C in the item above) would be worth the same amount (say, for example, 2 points each; so the entire item above would be worth a total of 6 points). **But above all, NOTE: Just an <u>answer</u> (T/F/N) <u>alone</u> is not awarded <u>any</u> points without the accompanying explanation.***

A Sample Free-Form Item

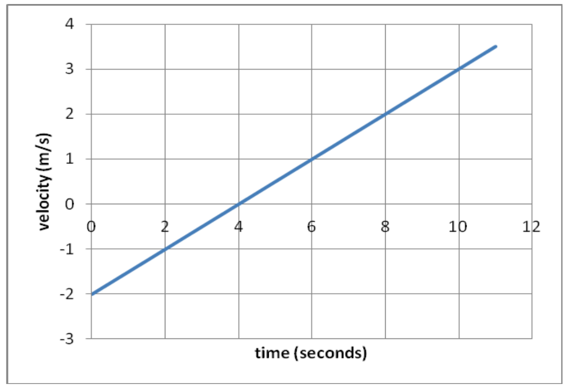
Here is how an item like this would appear in a Prep set or on an exam. You are being asked to provide one or more answers (either words, mathematical expressions/equations, or numerical values)—and the number of questions (parts) may vary; below, for example is a three- part item. These do not require an ODAVEST sort of solution (see next item); these are generally up to you to answer and justify in any way you choose. But the one rule overall (and the question may also remind you) is, as always: ***Along with your answer, you must justify your reasoning, in writing, with your own words and/or mathematical work.***

4. A velocity-time graph is shown for some moving object.

a. What is this object's displacement over the first 8.0 seconds? *Explain how you know.*

b. What is this object's acceleration at 6.0 seconds? *Explain how you know.*

c. What is this object's velocity at 4.0 seconds? What is happening at this moment? *Explain your answers.*



Below is one example of a completely correct set of answers. Again, the wording of the explanations will be different for each person. So these could be more succinct (so clear phrases rather than complete sentences are fine) and still considered acceptable. The versions given below include more than enough discussion in order to show you the full range of what could be offered to explain this problem's answers.

4. a. 0 m

On a velocity-time (v-t) graph, the displacement of the object is the area under the curve: $\Delta x = \int_{t_i}^{t_f} v(t)\, dt$

In this case, from the graph, we can see that v is a simple linear function of t: $v(t) = (1/2)t - 2$

Integrating, we get $\Delta x = \left[(1/4)t^2 - 2t\right]_0^8 = 0$

Alternatively: With the simple and symmetric geometry of the graph, the area under the curve is just the sum of the areas of two equal triangles (the one below the t-axis a "negative" area); hence a "net" area of zero.

b. 0.5 m/s²

On a v-t graph, the acceleration, $a(t)$, of the object at any point is the slope of the curve at that point: $a(t) = dv/dt$

In this case, from the graph, we can see that v is a simple linear function of t: $v(t) = (1/2)t - 2$

Differentiating, we get $a(t) = dv/dt = 1/2$.

Alternatively: With the simple geometry of the graph, we can see that the "rise-over-run" slope value of the curve at every point is simply 1/2.

c. 0 m/s; the object is reversing its direction of motion at that moment.

On a v-t graph, the velocity, $v(t)$, of the object at any point is simply the value of the graph at that point. Looking at the graph, we see that the value of v at $t = 4.0$ s, is zero (the graph intersects the t-axis at that moment). Since the value of the graph, v, was negative (i.e. the object was moving in the negative direction) before $t = 4.0$ s, but then v is positive (i.e. the object is moving in the positive direction) after $t = 4.0$ s, that moment of zero velocity ($t = 4.0$ s) represents the point of direction reversal.

*In any Free-Form item, each part might be worth a different point value (say, for the example above, 2 points for **a**, 3 points for **b**, and 4 points for **c**; so the entire item above would be worth a total of 9 points).* ***But again, NOTE: An answer alone is not awarded any points without the accompanying explanation.***

A Sample ODAVEST Problem

*Here is how a problem like this could appear in a Prep set, HW set or exam. In your solution(s), you should employ whatever parts of the full ODAVEST procedure that are asked for in the problem—**generally only the V-E-S portions are asked for.** However, to fully illustrate all possible portions—and reflect real-life technical solutions—the following asks for the full ODAVEST protocol.*

In Usain Bolt's 2009 world record time (9.58 s) for the 100-m sprint, his top speed was clocked at 27.794 mph. To break his own record in a future race (assuming the same top speed), what minimum acceleration must he achieve? **Solve this problem in general—that is, without using actual numbers—using the full ODAVEST protocol**.

*Here is one example of a completely correct solution, which requires inclusion of all seven parts of the problem-solving procedure we have dubbed **ODAVEST**. Note that more discussion (such as the rationale, or the possible alternative or additional assumptions) is included here for illustrative purposes than you'd have to include in your explanations; **the portions in bold would be sufficient**. But plan to use ample space; on an exam, you would be given two or three full pages of space for the V-E-S portions:*

*Re-state the **O**bjective in your own words:* Find the minimum acceleration magnitude required (starting from rest) for a competitive sprinter to attain a known top speed and to travel a known required distance in <u>less than</u> a known amount of time.

*List the **D**ata—all known values that you're explicitly given:*

v_{start} is the starting speed of the runner (= 0; the runner is starting from rest).

v_{max} is the known top speed of the runner.

Δx_{total} is the known required total distance traveled by the runner.

Δt_{total} is a known time interval just greater than the total duration of the run.

a_{min} is the minimum acceleration necessary for the runner to travel the distance Δx_{total} in a time less than Δt_{total}.

*List all additional **A**ssumptions you're also making:*

<u>Object model</u>: **We will model the runner as a <u>particle</u>.** That is, we're not really taking into account the various motions of his arms, legs, etc.—nor are we concerned with the fact that his head (nose), which is what typically triggers the clock stoppage at the finish line, actually starts ahead of the starting line (it's just his feet that must start completely behind that line).

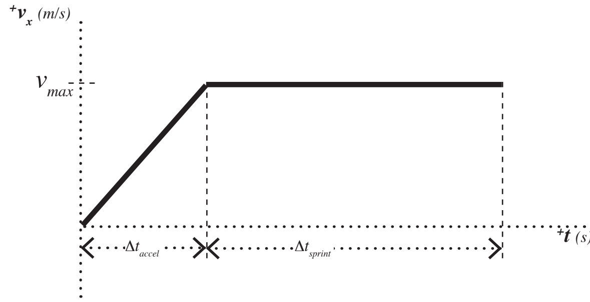
<u>Conditions</u>: We will assume that the distance is a **straight line**. (Motion at a given speed along a curved path requires more total acceleration than does the same speed along a straight path.)

<u>The motion</u>: We will assume that the runner does not "jump the gun but also that his reaction time to the starter is negligible. Thus: **His motion begins precisely at $t = 0$.**

If the runner were superhuman, he could go from 0 to v_{max} in an instant—essentially, sprint the whole distance, Δx_{total}, at his top speed, v_{max}. (And thus, the lower mathematical limit to Δt_{total} is $\Delta x_{total}/v_{max}$). But that would be an infinite acceleration rate—clearly unrealistic, but it's the <u>upper</u> mathematical limit to possible values of a here. A lower mathematical limit? That's the a value that would let the runner attain his v_{max} just as he crosses the finish line—also unrealistic: Clearly, to be competitive—minimize his race time—*a sprinter tries to reach his top speed as quickly as he can*. Thus:

We will model the motion as having two parts: an acceleration phase with a single constant acceleration value, a_{min}, followed by a sprint phase, at a constant v_{max} (thus $a = 0$) for the remainder of the race.

*Draw a **V**isual representation of the physics of the situation—something from which you can produce a mathematical model or equation(s)—a visual aid such as a graph or diagram.*



*Write the **E**quation(s) you need to get to the objective (and be sure to define all variables with meaningful names).*

Acceleration phase: $\Delta x_{accel} = v_{i.accel}(\Delta t_{accel}) + (1/2)a_{min}(\Delta t_{accel})^2$

$v_{f.accel} = v_{i.accel} + a_{min}\Delta t_{accel}$

Simplified: **I.** $\Delta x_{accel} = (1/2)a_{min}(\Delta t_{accel})^2$ (because $v_{i.accel} = 0$)

II. $v_{max} = a_{min}\Delta t_{accel}$ (because $v_{i.accel} = 0$ and $v_{f.accel} = v_{max}$)

Sprint phase: $\Delta x_{sprint} = v_{i.sprint}(\Delta t_{sprint}) + (1/2)a_{sprint}(\Delta t_{sprint})^2$

Simplified: **III.** $\Delta x_{sprint} = v_{max}(\Delta t_{sprint})$ (because $a_{sprint} = 0$, and $v_{i.sprint} = v_{max}$)

Entire race: **IV.** $\Delta x_{accel} + \Delta x_{sprint} = \Delta x_{total}$

V. $\Delta t_{accel} + \Delta t_{sprint} = \Delta t_{total}$

*Explain how to **S**olve the equation(s) for the objective.*

Substitute **I** and **III** into **IV**. Call this result **VI**.

Solve **II** for Δt_{accel}. Substitute that result into **V** and **VI**.

Solve **V** for Δt_{sprint}. Substitute that result into **VI**.

Solve **VI** for a_{min}.

*Explain the dimensions of your solution and also how you would **T**est it for reasonableness by predicting how it would change if just one of the known values were changed.*

Dimensions: The solution would need dimensions of *distance/time*^{2}, ***which is acceleration***.

Dependencies: A greater Δt_{total} (a slower current world-record time), with the race distance and maximum sprint speed left unchanged, would require a lesser a_{min}.

A larger Δx_{total} (increasing the distance of the race), with the same top speed and same finishing time, would require a greater a_{min}.

A larger v_{max} (increasing the runner's sprint speed), while still demanding the same finishing time for the same length race, would require a lesser a_{min}.