Static Fields Homework 8

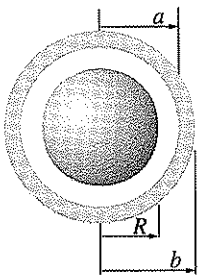
Due 4/26/19 @ 4:00 pm

PRACTICE:

REQUIRED:

1. (2 pts each)

   A metal sphere of radius R, carrying charge q is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as shown below). The shell carries no net charge.



   (a) Find the surface charge density σ at R, at a, and at b.

   (b) Find E_r, the radial component of the electric field and plot it as a function of r.

   (c) Find the potential at the center of the sphere, using infinity as the reference point.

   (d) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (the same as infinity). How do your answers to a), b), and c) change?

2. (2 pts each)

   Three charges are situated at the corners of a square (side s). Two have charge $-q$ and are located on opposite corners. The third has charge $+q$ and is opposite an empty corner.

   (a) How much work does it take to bring in another charge, $+q$, from far away and place it at the fourth corner?

   (b) How much work does it take to assemble the whole configuration of four charges?

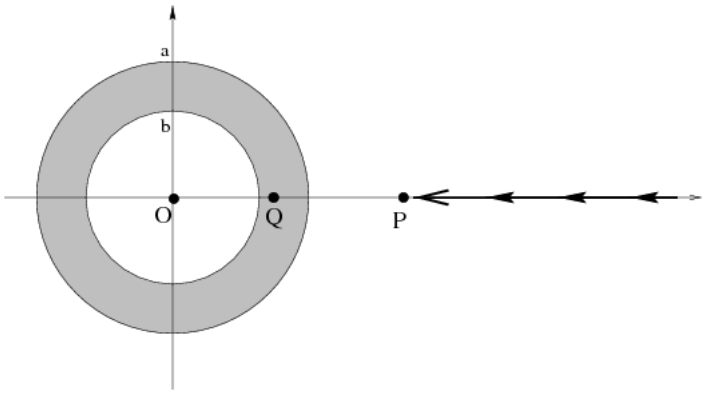
3. (2, 2, 2, 4, 2 pts)

   The gravitational field due to a spherical shell of mass is given by:

$$\vec{g} = \begin{cases} 0 & r < b \\ -\frac{4}{3}\pi\rho G\left(r - \frac{b^3}{r^2}\right)\hat{r} & b < r < a \\ -\frac{4}{3}\pi\rho G\left(\frac{a^3-b^3}{r^2}\right)\hat{r} & a < r \end{cases}$$

where b is the inside radius of the shell, a is the outside radius of the shell, and ρ is the constant mass density.

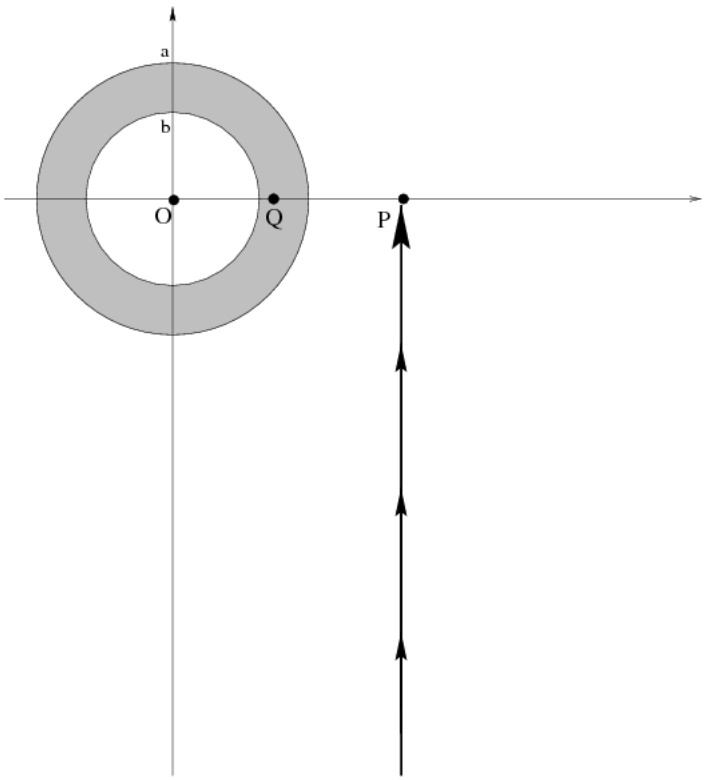
(a) Using an explicit line integral, calculate the work required to bring a test mass, of mass m_0, from infinity to a point P, which is a distance c (where $c > a$) from the center of the shell.



(b) Using an explicit line integral, calculate the work required to bring the test mass along the same path, from infinity to the point Q a distance d (where $b < d < a$) from the center of the shell.

(c) Using an explicit line integral, calculate the work required to bring the test mass along the same radial path from infinity all the way to the center of the shell.

(d) Using an explicit line integral, calculate the work required to bring in the test mass along the path drawn below, to the point P of question a. Compare the work to your answer from question a.



(e) What is the work required to bring the test mass from infinity along the path drawn below to the point P of question a. Explain your reasoning.