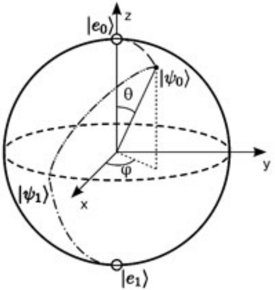
Quantum Spookiness

EPR paradox and beyond



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February 1, 2013

Albert Einstein and "spooky action at distance"

- Did not like the **probabilistic** nature of quantum mechanics.
- Physical object have an abjective reality **independent** of the measurement.
- Due to **superposition** principle in quantum mechanics this is not the case: before the measurement we do not know in what state is our system.
- Einstein: quantum mechanics is **incomplete** description of a reality.

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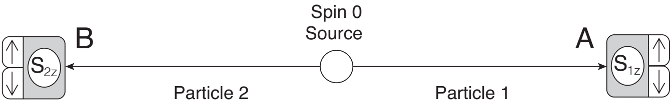

- Alice and Bob measure the spin component of their particles.
- Both have 50% chance to get either spin $|\uparrow\rangle$ or $|\downarrow\rangle$.
- Hovewer, because of the "spookiness" of quantum mechanics, whenever Alice measures spin $|\uparrow\rangle$, Bob with certainty measures spin $|\downarrow\rangle$!
- EPR conclusin: because we can predict the result of an experiment with certainty, the spin direction of a particle must be determined before measurement (**local hidden variable theory**).

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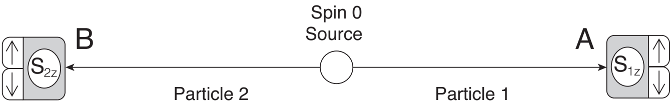

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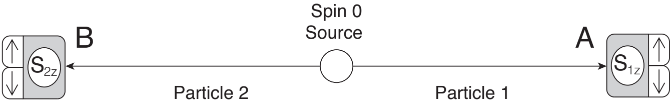

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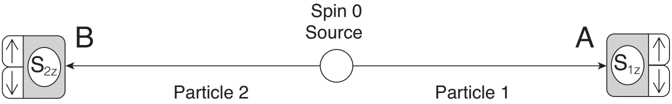

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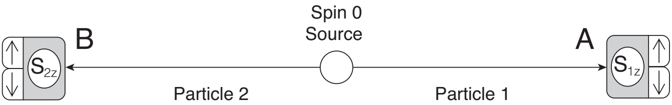

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We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

Schrödinger response

I would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.

- Particles that interact and then separate leading to the EPR-like results are called **entangled** (Verschränkung).

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Tensor Product Space

Postulate 1

The state of a quantum system, including all the information you can know about it, is represented mathematically by a normalized ket $|\psi\rangle$ - a quantum state vector.

- All quantum state vectors belong to a complete vector space (called a **Hilbert space**). Completness implies that each ket can be represented as a **superposition** of a basis kets:

$$|\psi\rangle \doteq \alpha|+\rangle + \beta|-\rangle$$

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Tensor Product Space

- A quantum state of two-particle system is represented as a tensor product of kets representing each particle, i.e,

$$|\psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_2 = |\psi\rangle_1 |\psi\rangle_2$$

- An arbitrary state can on a tensor space can be represented as:

$$|\Psi\rangle = \sum_{i,j=1}^{2} |\psi\rangle_i |\psi\rangle_j.$$

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Properties of the Tensor Product Space

- An operator acting on a tensor produc state acts only on its "own" ket. For instance, for $|\Psi\rangle \doteq |+\rangle_1 |-\rangle_2$:

$$\mathcal{S}_{1z}|\Psi\rangle = \Big(\mathcal{S}_{1z}|+\rangle_1\Big)|-\rangle_2 = \frac{\hbar}{2}|+\rangle_1|-\rangle_2$$

while

$$\mathcal{S}_{2z}|\Psi\rangle = |+\rangle_1\Big(\mathcal{S}_{1z}|-\rangle_2\Big) = -\frac{\hbar}{2}|+\rangle_1|-\rangle_2.$$

- An **inner product** is defined as follows:

$$\langle\Psi|\Psi\rangle \doteq \Big({}_1\langle+|{}_2\langle-|\Big)\Big(|+\rangle_1|-\rangle_2\Big) = \Big({}_1\langle+|+\rangle_1\Big)\Big({}_2\langle-|-\rangle_2\Big) = 1$$

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Superposition vs. Mixture ($\{|+\rangle, |-\rangle\}$)

Superposition

$$|\psi\rangle \doteq \frac{1}{\sqrt{2}}\left(|+\rangle_1|+\rangle_2 + |-\rangle_1|-\rangle_2\right)$$

Mixture

50 % of $|\psi_+\rangle \doteq |+\rangle_1|+\rangle_2$,
50 % of $|\psi_-\rangle \doteq |-\rangle_1|-\rangle_2$

$$|+\rangle_1\langle+|\psi\rangle = \frac{1}{\sqrt{2}}|+\rangle_1|+\rangle_2$$

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$$
\begin{aligned}
|\psi\rangle &\doteq \frac{1}{\sqrt{2}}\left(|+\rangle_1|+\rangle_2 + |-\rangle_1|-\rangle_2\right) \\
&= \frac{1}{2\sqrt{2}}\Big(\left(|+_x\rangle_1 + |-_x\rangle_1\right)\left(|+_x\rangle_1 + |-_x\rangle_1\right) \\
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Mixture

50 % of $|\psi_+\rangle \doteq |+\rangle_1 |+\rangle_2$ and 50 % of $|\psi_-\rangle \doteq |-\rangle_1 |-\rangle_2$

$|\psi_+\rangle \rightarrow$ 25% of: $|+_x\rangle_1|+_x\rangle_2$, $|+_x\rangle_1|-_x\rangle_2$, $|-_x\rangle_1|+_x\rangle_2$, $|-_x\rangle_1|-_x\rangle_2$

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$$|\psi\rangle \doteq \frac{1}{\sqrt{2}}\left(|+_x\rangle_1|+_x\rangle_2 + |+_x\rangle_1|-_x\rangle_2\right)$$

$$|+_x\rangle_1\langle_x+|\psi\rangle = \frac{1}{\sqrt{2}}|+_x\rangle_1|+_x\rangle_2$$

$$\mathcal{P}_{1,x+} = \frac{1}{2} \wedge \mathcal{P}_{2,x+} = 1$$

$$|-_x\rangle_1\langle_x-|\psi\rangle = \frac{1}{\sqrt{2}}|-_x\rangle_1|-_x\rangle_2$$

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Superposition vs. Mixture $(\{|+_x\rangle, |-_x\rangle\})$

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$$|\psi\rangle \doteq \frac{1}{\sqrt{2}}\left(|+_x\rangle_1 |+_x\rangle_2 + |+_x\rangle_1 |-_x\rangle_2\right)$$

$$|+_x\rangle_1 \langle x+|\psi\rangle = \frac{1}{\sqrt{2}}|+_x\rangle_1 |+_x\rangle_2$$

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Superposition vs. Mixture ($\{|+_x\rangle, |-_x\rangle\}$)

Mixture

25 % of: $|+_x\rangle_1|+_x\rangle_2$, $|+_x\rangle_1|-_x\rangle_2$, $|-_x\rangle_1|+_x\rangle_2$, $|-_x\rangle_1|-_x\rangle_2$

- There is $\frac{1}{2}$ probability to measure $|+_x\rangle_1$, however:

$$|+_x\rangle_1\langle +_x|\psi\rangle = \begin{cases} |+_x\rangle_1|+_x\rangle_2 \\ |+_x\rangle_1|-_x\rangle_2 \end{cases} \Rightarrow \mathcal{P}_{2,+/-} = \frac{1}{2}!$$

- Similarly there is $\frac{1}{2}$ probability to measure $|-_x\rangle_1$, but:

$$|-_x\rangle_1\langle -_x|\psi\rangle = \begin{cases} |-_x\rangle_1|+_x\rangle_2 \\ |-_x\rangle_1|-_x\rangle_2 \end{cases} \Rightarrow \mathcal{P}_{2,+/-} = \frac{1}{2}!$$

Bell's inequalities (classicallcy)

- Suppose we measured $N = \sum_{i=1}^{8} N_i$ particles:

Population	Particle 1	Particle 2
N_1	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$
N_2	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$
N_3	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_4	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$
N_5	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$
N_6	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$
N_7	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_8	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$

9 possible measurement directions:

$$\hat{\mathbf{a}}\hat{\mathbf{a}},\ \hat{\mathbf{a}}\hat{\mathbf{b}},\ \hat{\mathbf{a}}\hat{\mathbf{c}},\ \hat{\mathbf{b}}\hat{\mathbf{a}},\ \hat{\mathbf{b}}\hat{\mathbf{b}},\ \hat{\mathbf{b}}\hat{\mathbf{c}},\ \hat{\mathbf{c}}\hat{\mathbf{a}},\ \hat{\mathbf{c}}\hat{\mathbf{b}},\ \hat{\mathbf{c}}\hat{\mathbf{c}}$$

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N_7	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_8	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$

For type 1 and 8 particles:

$$\mathcal{P}_{same} = 0$$

$$\mathcal{P}_{opp} = 1$$

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N_3	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_4	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$
N_5	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$
N_6	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$
N_7	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$
N_8	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$

For type $2-7$ particles:

$$\mathcal{P}_{same} = \frac{5}{9}$$

$$\mathcal{P}_{opp} = \frac{4}{9}$$

Bell's inequalities (classicallcy)

- We can bound probabilities of recording the same and opposite result as follows:

$$\mathcal{P}_{same} = \frac{1}{N}\left(0 \cdot N_1 + \frac{4}{9}(N_2 + N_3 + N_4 + N_5 + N_6 + N_7) + 0 \cdot N_8\right) \leq \frac{4}{9}$$

$$\mathcal{P}_{opp} = \frac{1}{N}\left(1 \cdot N_1 + \frac{5}{9}(N_2 + N_3 + N_4 + N_5 + N_6 + N_7) + 1 \cdot N_8\right) \geq \frac{5}{9}$$

Quantum Bell's inequalities

$$|\psi\rangle \doteq \frac{1}{\sqrt{2}}\Big(|+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2\Big)$$

$$\mathcal{P}_{++} = \Big|\big({}_1\langle+|_{2\hat{\mathbf{n}}}\langle+|\big)|\psi\rangle\Big|^2 = \frac{1}{2}\sin^2\frac{\theta}{2}$$

$$\mathcal{P}_{same} = \mathcal{P}_{++} + \mathcal{P}_{--} = \Big|\big({}_1\langle+|_{2\hat{\mathbf{n}}}\langle+|\big)|\psi\rangle\Big|^2 + \Big|\big({}_1\langle-|_{2\hat{\mathbf{n}}}\langle-|\big)|\psi\rangle\Big|^2$$

and

$$\mathcal{P}_{+-} = \Big|\big({}_1\langle+|_{2\hat{\mathbf{n}}}\langle-|\big)|\psi\rangle\Big|^2 = \frac{1}{2}\cos^2\frac{\theta}{2}$$

$$\mathcal{P}_{opp} = \mathcal{P}_{+-} + \mathcal{P}_{-+} = \Big|\big({}_1\langle+|_{2\hat{\mathbf{n}}}\langle-|\big)|\psi\rangle\Big|^2 + \Big|\big({}_1\langle-|_{2\hat{\mathbf{n}}}\langle+|\big)|\psi\rangle\Big|^2$$

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QUANTUM BELL'S INEQUALITIES

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$$\mathcal{P}_{same} \leq \frac{4}{9}$$

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ASPECT'S EXPERIMENT (1981)

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

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(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

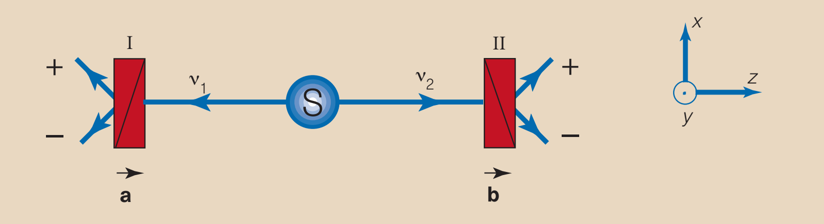
PACS numbers: 03.65.Bz, 32.50.+d, 32.80.Kf

Since the development of quantum mechanics, there have been repeated suggestions that its statistical features possibly might be described by an underlying deterministic substructure, a quantum state representing a statistical ensemble of "hidden-variable states." In 1965 Bell[1] showed that any such "hidden-variable substruc-

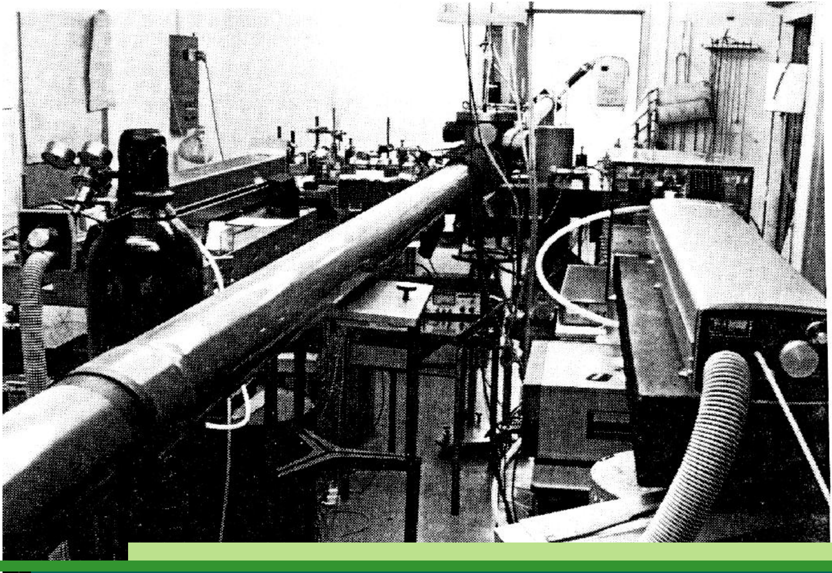
but requires similar assumptions.

Pairs of low-energy photons emitted in certain atomic radiative cascades are candidates for better tests.[2] With a reasonable assumption about the detector efficiencies,[2,3] the actual experiments constitute a valuable test of local realistic theories via Bell's theorem. So far

Aspect's experiment (1981)



Aspect's experiment (1981)



Conclusion

Einstein:

"I, at any rate, am convinced that He (God) does not throw dice."

Bohr:

"Einstein, don't tell God what to do."

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