

Spin 1/2 reference sheet

$$\langle +|+\rangle = \langle -|-\rangle = 1$$

$$\langle +|-\rangle = \langle -|+\rangle^* = 0$$

$$|+\rangle_x = \frac{1}{\sqrt{2}}[|+\rangle + |-\rangle]$$

$$|+\rangle_y = \frac{1}{\sqrt{2}}[|+\rangle + i|-\rangle]$$

$$\hat{\mathbf{n}} = \hat{\mathbf{i}} \sin \theta \cos \phi + \hat{\mathbf{j}} \sin \theta \sin \phi + \hat{\mathbf{k}} \cos \theta$$

$$|+\rangle_n = \cos \frac{\theta}{2} |+\rangle + \sin \frac{\theta}{2} e^{i\phi} |-\rangle$$

$$|-\rangle_x = \frac{1}{\sqrt{2}}[|+\rangle - |-\rangle]$$

$$|-\rangle_y = \frac{1}{\sqrt{2}}[|+\rangle - i|-\rangle]$$

$$|-\rangle_n = \sin \frac{\theta}{2} |+\rangle - \cos \frac{\theta}{2} e^{i\phi} |-\rangle$$

$$S_z |+\rangle = +\frac{\hbar}{2} |+\rangle$$

$$S_x |+\rangle_x = +\frac{\hbar}{2} |+\rangle_x$$

$$S_y |+\rangle_y = +\frac{\hbar}{2} |+\rangle_y$$

$$S_z |-\rangle = -\frac{\hbar}{2} |-\rangle$$

$$S_x |-\rangle_x = -\frac{\hbar}{2} |-\rangle_x$$

$$S_y |-\rangle_y = -\frac{\hbar}{2} |-\rangle_y$$

Using the S_z representation: (\doteq means “is represented by”)

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S_z \doteq \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y \doteq \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_n = S_x \sin \theta \cos \phi + S_y \sin \theta \sin \phi + S_z \cos \theta$$

$$S_n \doteq \frac{\hbar}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}$$

Pauli Matrices:

$$\vec{S} \doteq \frac{\hbar}{2} \vec{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x |\pm\rangle_x = \pm |\pm\rangle_x$$

$$\sigma_y |\pm\rangle_y = \pm |\pm\rangle_y$$

$$\sigma_z |\pm\rangle = \pm |\pm\rangle$$

Spin 1 reference sheet

$$|1\rangle_x = \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$$

$$|1\rangle_y = \frac{1}{2}|1\rangle + \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|0\rangle_x = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

$$|0\rangle_y = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|-1\rangle$$

$$|-1\rangle_x = \frac{1}{2}|1\rangle - \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}|-1\rangle$$

$$|-1\rangle_y = \frac{1}{2}|1\rangle - \frac{i}{\sqrt{2}}|0\rangle - \frac{1}{2}|-1\rangle$$

$$|1\rangle_n = \frac{1 + \cos \theta}{2} e^{-i\phi} |1\rangle + \frac{\sin \theta}{\sqrt{2}} |0\rangle + \frac{1 - \cos \theta}{2} e^{i\phi} |-1\rangle$$

$$|0\rangle_n = -\frac{\sin \theta}{\sqrt{2}} e^{-i\phi} |1\rangle + \cos \theta |0\rangle + \frac{\sin \theta}{\sqrt{2}} e^{i\phi} |-1\rangle$$

$$|-1\rangle_n = \frac{1 - \cos \theta}{2} e^{-i\phi} |1\rangle - \frac{\sin \theta}{\sqrt{2}} |0\rangle + \frac{1 + \cos \theta}{2} e^{i\phi} |-1\rangle$$

$$S_z |1\rangle = \hbar |1\rangle \quad S_z |0\rangle = 0 \quad S_z |-1\rangle = -\hbar |-1\rangle$$

Using the S_z representation: (\doteq means “is represented by”)

$$|1\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|0\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|-1\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$S_z \doteq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S_y \doteq \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$S_n \doteq \hbar \begin{pmatrix} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta e^{-i\phi} & 0 \\ \frac{1}{\sqrt{2}} \sin \theta e^{i\phi} & 0 & \frac{1}{\sqrt{2}} \sin \theta e^{-i\phi} \\ 0 & \frac{1}{\sqrt{2}} \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}$$