

PH425 Spins Homework 1

Due 1/13/16 @ 4 pm

PRACTICE:

1. (Quiz)

- Use Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$ and its complex conjugate to find formulas for $\sin \theta$ and $\cos \theta$. In your physics career, you will often need to read these formula "backwards," i.e. notice one of these combinations of exponentials in a sea of other symbols and say, "Ah ha! that is $\cos \theta$." So, pay attention to the result of the homework problem!
- Show that $e^{2i\theta} + e^{-4i\theta} = 2e^{-i\theta} \cos(3\theta)$ by manipulating the left hand side until it looks like the right hand side of the equation. This calculation is similar to many calculations you will make in the course of your careers related to time or spatial dependence of quantum systems. So, pay attention to the methods of this homework problem!

REQUIRED:

- Using the formula

$$\vec{\mu} = \frac{1}{2} \int \vec{r} \times \vec{J}(\vec{r}) d\tau$$

find the dipole moment of a wire carrying current I and bent in the shape of a square with sides of length L .

- Using the formula

$$\vec{\mu} = \frac{1}{2} \int \vec{r} \times \vec{J}(\vec{r}) d\tau$$

find the dipole moment of a charged spinning disk with constant charge density, total charge Q , radius R , and period T .

- Consider a square wire with sides of length L carrying current I . The normal to the plane of the wire loop is at an angle θ with respect to a constant magnetic field \vec{B} . Take the direction of the magnetic field to be \hat{z} and the high side of the wire to be at constant positive values of x .
 - Find the force on each of the sides of the wire.
 - Find the total force on the wire.
 - Find the torque on each side of the wire.
 - Find the total torque on the wire.

(e) Show that the energy of the wire in the external magnetic field is given by:

$$H = -\vec{\mu} \cdot \vec{B}$$

(f) Given that the energy of the wire in the external magnetic field is given by:

$$H = -\vec{\mu} \cdot \vec{B}$$

What configuration has zero energy? minimum energy? maximum energy?

5. Consider the three quantum states:

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{5}}|+\rangle - \frac{2}{\sqrt{5}}|-\rangle$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}|+\rangle + i\frac{e^{i\pi/4}}{\sqrt{2}}|-\rangle$$

Use bra-ket notation (not matrix notation) to solve the following problems.

- (a) For each of the $|\psi_i\rangle$ above, find the normalized vector $|\phi_i\rangle$ that is orthogonal to it.
- (b) Calculate the inner products $\langle\psi_i|\psi_j\rangle$ for i and $j = 1, 2, 3$.