PH425 Spins Homework 2

Due 1/15/16 @ 4 pm

PRACTICE:

1. **Quiz**

   A beam of spin-$\frac{1}{2}$ particles is prepared in the state:

$$|\psi\rangle = \frac{2}{\sqrt{13}}|+\rangle + i\frac{3}{\sqrt{13}}|-\rangle$$

   (a) What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

REQUIRED:

2. A beam of spin-$\frac{1}{2}$ particles is prepared in the state:

$$|\psi\rangle = \frac{2}{\sqrt{13}}|+\rangle + i\frac{3}{\sqrt{13}}|-\rangle$$

   (a) What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

   (b) What are the possible results of a measurement of the spin component S_x, and with what probabilities would they occur?

   (c) Plot histograms of the predicted measurement results from parts (a) and (c).

3. Consider the three quantum states:

$$|\psi_1\rangle = \frac{4}{5}|+\rangle + i\frac{3}{5}|-\rangle$$
$$|\psi_2\rangle = \frac{4}{5}|+\rangle - i\frac{3}{5}|-\rangle$$
$$|\psi_3\rangle = -\frac{4}{5}|+\rangle + i\frac{3}{5}|-\rangle$$

   (a) For each of the $|\psi_i\rangle$ above, calculate the probabilities of spin component measurements along the x, y, and z-axes.

   (b) Use your results from (a) to comment on the importance of the overall phase and of the relative phases of the quantum state vector.

4. Using the Spins simulation, choose the `Spin-1` case under the `Design` menu. Set up an experiment for two successive meaurements of spin projections.

(a) Measure the probability that a state which starts out with z-component of spin equal to $\hbar$ ends up with z-component of spin equal to $\hbar$ after the z-component of spin is measured. Write your statement in bra-ket language.

(b) Measure the probability that a state which starts out with z-component of spin equal to $\hbar$ ends up with z-component of spin equal to zero after the z-component of spin is measured. Write your statement in bra-ket language. What does this probability tell you about the z basis?

(c) Measure the probability that a state which starts out with x-component of spin equal to zero ends up with z-component of spin equal to zero after the z-component of spin is measured. Write your statement in bra-ket language. What does this probability tell you about the x and z bases?

(d) Use your simulation to find the value of $|\langle 1| - 1\rangle_x|^2$. State in words what the measured quantity represents. Compare your "measured" value to a theoretical value computed from the Spin Reference Sheet.

5. Consider a quantum system described by an orthonormal basis $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$. The system is initially in a state:

$$|\psi_{\text{in}}\rangle = \frac{i}{\sqrt{3}}|a_1\rangle + \sqrt{\frac{2}{3}}|a_2\rangle$$

Find the probability that the system is measured to be in the final state:

$$|\psi_{\text{out}}\rangle = \frac{1+i}{\sqrt{3}}|a_1\rangle + \frac{1}{\sqrt{6}}|a_2\rangle + \frac{1}{\sqrt{6}}|a_3\rangle$$