PH425 Spins Homework 4

Due 1/22/16 @ 4 pm

REQUIRED:

1. Show that the kets $|+\rangle_y$ and $|-\rangle_y$ defined by

$$|+\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|-\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

form an orthonormal basis for the vector space of two-component complex vectors, i.e.

   (a) Show that $|+\rangle_y$ and $|-\rangle_y$ are normalized.

   (b) Show that $|+\rangle_y$ and $|-\rangle_y$ are orthogonal.

   (c) Show that $|+\rangle_y$ and $|-\rangle_y$ are complete, i.e. that any vector in the vector space can be written as a linear combination of these two vectors.

2. Show that the matrix

$$A \doteq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

is a linear operator on the space of all vectors, i.e.:

   (a) Show that

$$A\left(|v_1\rangle + |v_2\rangle\right) = A|v_1\rangle + A|v_2\rangle$$

   (b) Also show that

$$A\left(\lambda|v_1\rangle\right) = \lambda A|v_1\rangle$$

3. Using the Spin 1 version of the Spins simulation, find the probabilities for the projection of the spin onto all three standard axes for unknowns $|\psi_1\rangle$ and $|\psi_3\rangle$. Use these probabilities to write the unknowns in the z-basis.

4. If a beam of spin-3/2 particles is input to a Stern-Gerlach analyzer, there are four output beams whose deflections are consistent with magnetic moments arising from spin angular momentum components of $\frac{3}{2}\hbar$, $\frac{1}{2}\hbar$, $-\frac{1}{2}\hbar$, and $-\frac{3}{2}\hbar$. For a spin-3/2 system:

   (a) Write down the eigenvalue equations for the S_z operator.

   (b) Write down the matrix representation of the S_z eigenstates.

   (c) Write down the matrix representation of the S_z operator.

   (d) Write down the eigenvalue equations for the S^2 operator.

(e) Write down the matrix representation of the S^2 operator.

CHALLENGE:

5. Using the Spin 1 version of the Spins simulation, find the probabilities for the projection of the spin onto all three standard axes for the unknown $|\psi_2\rangle$. Use these probabilities to write the unknown in the z-basis.