PH425 Spins Homework 5

Due 1/27/16 at 4 pm

REQUIRED:

1. Consider a quantum system with an observable A that has three possible measurement results: a_1, a_2, and a_3.

   (a) Write down the three kets $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$, corresponding to these possible results, using matrix notation.

   (b) The system is prepared in the state:

   $$|\psi\rangle = 1|a_1\rangle - 2|a_2\rangle + 5|a_3\rangle$$

   Staying in bra-ket notation, calculate the probabilities of all possible measurement results of the observable A. Plot a histogram of the predicted measurement results.

   (c) In a different experiment, the system is prepared in the state:

   $$|\psi\rangle = 2|a_1\rangle + 3i|a_2\rangle$$

   Write this state in matrix notation and calculate the probabilities of all possible measurement results of the observable A. Plot a histogram of the predicted measurement results.

2. Consider a three-dimensional state space. In the basis defined by three orthonormal kets $|1\rangle$, $|2\rangle$, and $|3\rangle$, the operators A and B are represented by:

   $$A \doteq \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \qquad B \doteq \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}$$

   where all the matrix elements are real.

   (a) Do the operators A and B commute?

   (b) Find the eigenvalues and normalized eigenvectors of both operators.

   (c) Assume the system is initially in the state $|2\rangle$. Then the observable corresponding to the operator B is measured. What are the possible results of this measurement and the probabilities of each result? After this measurement, the observable corresponding to the operator A is measured. What are the possible results of this measurement and the probabilities of each result?

   (d) How are questions (a) and (c) above related?

3. Consider a spin 1 interferometer which prepares the state as $|\hbar\rangle$, then sends this state through an S_x apparatus and then an S_z apparatus. Measure the relative probabilities after the final Stern-Gerlach analyzer for the seven possible cases where one beam, a pair of beams, or all three beams from the S_x Stern-Gernach analyzer were used. Compare the simulation results to theory. Make sure that, for the theory section, you explicitly discuss your choice of projection operators.

4. A beam of spin-1 particles is prepared in the state

$$|\psi\rangle = \frac{2}{\sqrt{29}}|1\rangle + i\frac{3}{\sqrt{29}}|0\rangle - \frac{4}{\sqrt{29}}|-1\rangle.$$

   (a) What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

   (b) What are the possible results of a measurement of the spin component S_x, and with what probabilities would they occur?

   (c) Plot histograms of the predicted measurement results from parts (a) and (b), and calculate the expectation values for both measurements.

5. Consider the state $|-1\rangle_y$ in a spin 1 system. Calculate the expectation values and uncertainties for measurements of S_x, S_y, and S_z. Draw a diagram of the vector model applied to this state and reconcile your quantum mechanical calculations with the classical results.