PH425 Spins Homework 6

Due 1/29/16 at 4 pm

REQUIRED:

1. Consider a spin-1/2 particle with a magnetic moment. At time $t = 0$, the state of the particle is $|\psi(t = 0)\rangle = |+\rangle$.

   (a) If the observable S_x is measured at time $t = 0$, what are the possible results and the probabilities of those results?

   (b) Instead of performing the above measurement, the system is allowed to evolve in a uniform magnetic field $\vec{B} = B_0\,\hat{y}$. Calculate the state of the system after a time t using the S_z basis.

   (c) At time t, the observable S_x is measured. What is the probability that a value $\hbar/2$ will be found?

   (d) Draw a schematic diagram of this experiment, similar to Fig. 3.2.

2. Consider a two-state quantum system with a Hamiltonian

$$H \doteq \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \tag{1}$$

   Another physical observable A is described by the operator

$$A \doteq \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} \tag{2}$$

   where a is real and positive. Let the initial state of the system be $|\psi(0)\rangle = |a_1\rangle$, where $|a_1\rangle$ is the eigenstate corresponding to the larger of the two possible eigenvalues of A. What is the frequency of oscillation of the expectation value of A? Compare this frequency to the Bohr frequency.

3. A quantum mechanical system starts out in the state:

$$|\psi(0)\rangle = C\,(3|a_1\rangle + 4|a_2\rangle) \tag{3}$$

   where $|a_i\rangle$ are the normalized eigenstates of the operator A corresponding to the eigenvalues a_i. In this $|a_i\rangle$ basis, the Hamiltonian of this system is represented by the matrix:

$$H \doteq E_0 \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \tag{4}$$

   (a) If you measure the energy of this system, what values are possible, and what are the probabilities of measuring those values?

(b) Find the state from the previous part as a function of time.

(c) Calculate the expectation value $\langle A \rangle$ of the observable A as a function of time. (This part of the problem is a Challenge. It is NOT required. If you can do this, you can do anything!)