

## Definitions for Analytic Functions

A function  $w(z)$  is *analytic* (or *regular* or *holomorphic* or *monogenic*) in a region of the complex plane if it has a derivative at **every** point in the region. (*Analytic at a point*, or *on a line*, means in an open region around that point or line.)

A *regular point* of  $w(z)$  is a point at which  $w(z)$  is analytic.

A *singular point* of  $w(z)$  is a point at which  $w(z)$  is **not** analytic.

An *isolated singular point* of  $w(z)$  is a point at which  $w(z)$  is **not** analytic but for which  $w(z)$  **is** analytic in a neighborhood of that point.

In the limit that  $z \rightarrow z_0$ , if a function  $w(z) \rightarrow \infty$  and  $(z - z_0)^n w(z) \rightarrow \infty$  for  $n \leq m$ , but  $(z - z_0)^m w(z)$  is finite, then  $w(z)$  is said to have a *pole of order  $m$*  at  $z_0$ .

A *branch point* is a singular point in a multiple-valued function such that the function is discontinuous when going around an arbitrarily small circle around this point in the domain.