Series Theorems for Analytic Functions

Theorem: (Taylor Series)
If $w(z)$ is analytic in a region, then it has derivatives of all orders and can be expanded in a Taylor series about any point in that region. The Taylor series will converge inside a circle which extends to the nearest sigular point in the complex plane.

Theorem: (Laurent Series)
Let C_1 and C_2 be two circles in the complex plane with center at z_0. If $w(z)$ is analytic in the region between the circles, then it can be expanded in a *Laurent series* of the form:

$$\begin{aligned} w(z) \;=\; & a_0 + a_1(z - z_0) + a_2(z - z_0)^2 + \ldots \\ & + \frac{b_1}{z - z_0} + \frac{b_2}{(z - z_0)^2} + \ldots \end{aligned}$$