

Uniqueness of Solutions of Linear ODEs

Consider the n^{th} order, linear, ordinary differential equation:

$$1y^{(n)} + a_{n-1}(x)y^{(n-1)} + \dots + a_0(x)y = \phi(x) \quad (1)$$

Theorem: Consider equation (1), together with the initial conditions

$$y(x_0) = c_0 \quad (2)$$

$$y'(x_0) = c_1 \quad (3)$$

$$\vdots \quad (4)$$

$$y^{(n-1)}(x_0) = c_{n-1} \quad (5)$$

If $\phi(x)$ and $a_j(x)$ are continuous $\forall j$ on some interval I containing x_0 , then the initial value problem has a unique solution throughout I .