General Solutions of Linear ODEs

Theorem: An nth order linear *homogeneous* differential equation $L(y) = 0$ always has n linearly independent solutions, $y_1, \ldots, y_n$. The general solution y_h is

$$y_h = \beta_1 y_1 + \beta_2 y_2 + \ldots + \beta_n y_n$$

where $\beta_1, \beta_2, \ldots, \beta_n$ are *arbitrary* constants.

Theorem: The *general* solution of the nth order linear *inhomogeneous* differential equation $L(y) = \phi(x)$ is

$$y = y_h + y_p$$

where y_h is the general solution of the homogeneous equation $L(y) = 0$ and y_p is *any* particular solution of the inhomogeneous equation.