# Orthogonal Polynomials

$P_\ell$ Legendre Polynomials $m=0$ } spherical coords
$P_\ell^m$ Associated Legendre Functions }

$L_n$ Laguerre Polynomials } radial Schrödinger in $\frac{1}{r}$ potential
$L_n^m$ Associated Laguerre Polynomials }

$H_n$ Hermite Polynomials QM Harmonic Oscillator

Jacobi (most general), Gegenbauer, Chebyshev

Bessel Functions – ~~angular~~ radial eqn is cylindrical coords (~~parabolic~~)

$$J_\nu(z), \; \begin{cases} Y_\nu(z) \\ N_\nu(z) \end{cases} = \frac{\cos(\nu\pi)\, J_\nu(z) - J_{-\nu}(z)}{\sin(\nu\pi)}$$

Neumann

$$H_\nu^{(1,2)}(z) = J_\nu(z) \pm i\, N_\nu(z)$$

Hankel

$I_\nu(z)$, $K_\nu(z)$ – modified or hyperbolic

$j_n(z)$, $y_n(z)$, $h_n^{(1,2)}(z)$ – spherical (half integer order)

vibrations in spherical coords

Fourier Series – particle in a box

$$\left(\frac{d^2}{dx^2} + n^2\right) f(x) = 0 \qquad \text{bc.} \quad f(0) = 0 = f(2\pi)$$

$$\lambda = n^2$$

$$w(x) = 1$$

$$f_n(x) = \sin nx$$

$$\int_0^{2\pi} \sin nx \, \sin mx \, dx = \pi \, \delta_{n,m}$$

$$g(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$\Rightarrow b_n = \frac{1}{\pi} \int_0^{2\pi} g(x) \sin nx \, dx$$

Legendre Polynomials – $\Theta$ eqn for $m=0$ in spherical coords

$$(1-z^2)\, y'' - 2zy' + \ell(\ell+1) y = 0 \qquad \text{b.c.} \quad P_\ell(1) = 1, \quad P_\ell(-1) = (-1)^\ell$$

$$\lambda = \ell(\ell+1)$$

$$w(z) = 1$$

$$y_n(z) = P_\ell(z)$$

$$\int_{-1}^{1} P_\ell(z) P_n(z) \, dz = \frac{2}{2\ell+1} \delta_{\ell n}$$

$$g(z) = \sum_{\ell=0}^{\infty} c_\ell P_\ell(z)$$

$$\Rightarrow c_\ell = \frac{2\ell+1}{2} \int_{-1}^{1} g(z) P_\ell(z) \, dz$$

# Laguerre Polynomials

b.c. polynomial on $0 \le x \le \infty$

Radial eqn from QM central force (atoms)

$$x y'' + (1-x) y' + n y = 0$$

To get in S-L form, multiply by $e^{-x}$

$$x e^{-x} y'' + (1-x) e^{-x} y' + n e^{-x} y = 0$$

$$\lambda = n$$

$$w(x) = e^{-x}$$

$$y_n = L_n(x)$$

Normalization is chosen nicely so this factor is 1

$$\int_0^\infty L_n(x) L_m(x) e^{-x} dx = \delta_{n,m}$$

$$g(x) = \sum_{n=0}^{\infty} c_n L_n(x) dx$$

$$c_n = \int_0^\infty g(x) L_n(x) e^{-x} dx$$