Definitions for Normed Vector Spaces

A set of objects (vectors) $\{\vec{u}, \vec{v}, \vec{w}, \ldots\}$ is said to form a *linear vector space* over the field of scalars $\{\lambda, \mu, \ldots\}$ (e.g. real numbers or complex numbers) if:

1. the set is closed, commutative, and associative under (vector) addition;
2. the set is closed, associative, and distributive under multiplication by a scalar;
3. there exists a *null vector* $\vec{0}$;
4. multiplication by the scalar identity 1 leaves the vector unchanged;
5. all vectors have a corresponding *negative vector*;

An *inner product* $\langle\vec{u}|\vec{v}\rangle$ is a generalization of the dot product with the following properties:

$$\langle\vec{u}|\vec{v}\rangle = \langle\vec{v}|\vec{u}\rangle^*$$

$$\langle\vec{u}|\lambda\vec{v} + \mu\vec{w}\rangle = \lambda\langle\vec{u}|\vec{v}\rangle + \mu\langle\vec{u}|\vec{w}\rangle$$