Wronskians

Definition: A set of n functions $y_1, \ldots, y_n$ on an interval I are *linearly dependent* if there exist constants $\alpha_1, \alpha_2, \ldots, \alpha_n$, not all zero, such that

$$\alpha_1 y_1 + \alpha_2 y_2 + \cdots + \alpha_n y_n = 0$$

Otherwise the functions are *linearly independent*.

Definition: If a set of n functions $y_1, \ldots, y_n$ on an interval I each have $n-1$ derivatives, then the determinant $W(y_1, \ldots, y_n)$, defined below, is called the *Wronskian* of the set of functions.

$$W(y_1, \ldots, y_n) \doteq \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & \vdots & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix} \tag{1}$$

Theorem: If $y_1, \ldots, y_n$ are solutions of $L(y) = 0$ on I, then they are linearly independent $\iff W(y_1, \ldots, y_n)$ is *not* identically zero on I.

Note: This theorem is only valid of the functions $y_1, \ldots, y_n$ are all solutions of the *same* n^{th} order linear ODE.