Due 4/4/16 4 pm

PRACTICE:

1. **Quiz 1**

   Express each of the following complex numbers in rectangular form, i.e. in the form $x + iy$ where x and y are real.

   (a) $3e^{2(1+i\pi)}$

   (b) $3e^{i\pi} + 3e^{-i\pi}$

   Express each of the following complex numbers in exponential form, i.e. in the form $re^{i\phi}$ where r and ϕ are real.

   (a) $\frac{2+3i}{1-i}$

   REQUIRED:

2. **Rectangular Form of Complex Functions**

   (a) Evaluate the complex number $w = \sin(3 + 2i)$ in the form $w = u + iv$ for u and v real. State whether it is pure real, pure imaginary, or complex. You do not need to find the numerical value of complicated **real** valued transcendental expressions, but do simplify them as much as possible. Check your answer using Mathematica or Maple.

   (b) Evaluate the complex function $w(z) = \sin(x + iy)$ in the form $w = u + iv$ for u and v real. State for which values of z the value of the function is pure real, pure imaginary, or complex. Check your answer using Mathematica or Maple.

3. **Large Powers of Complex Numbers**

   (a) Evaluate the complex number $w = \left(\frac{1-i}{\sqrt{2}}\right)^{40}$ in rectangular form, i.e. in the form $w = u + iv$ with u and v real.

   (b) Evaluate the complex number $w = (z)^{40}$ in rectangular form, i.e. in the form $w = u + iv$ with u and v real.

4. **Differential Equations Derivation of Euler's Formula**
   Show that the functions $\sin\theta$, $\cos\theta$, $e^{i\theta}$, and $e^{-i\theta}$ all solve the same differential equation. Use this result and appropriately chosen boundary conditions to "derive" Euler's formula.

5. **Complex Roots**
   Show that the sum of the three cube roots of 8 is zero. Make a sketch of the roots.

6. **Transcendental Functions of a Complex Variable**
   Express each of the following complex numbers in rectangular form, i.e. in the form
   $u + iv$ where u and v are real. State whether each number is pure real, pure imaginary,
   or complex. Also state whether it is single or multiple-valued. You do not need to
   find the numerical value of complicated **real**-valued transcendental expressions, but
   do simplify them as much as possible. Check your answers using a computer algebra
   system such as *Mathematica* or *Maple*.

   (a) $w = \ln(3 + 2i)$

   (b) $w = \sin^{-1} 3$

   (c) $w = i^{3+2i}$