PH461 Math Methods Capstone Homework 3

Due 4/18/16, 3:50 pm

PRACTICE:

1. **Quiz 3**

   Write out the first four nonzero terms in the series:

   (a)
$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

   Solution:

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{1}{n!} &= \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... \\ &= 1 + 1 + \frac{1}{2} + \frac{1}{6} + ...\end{aligned}$$

   (b)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

   Solution:

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} &= \frac{-1^1}{1!} + \frac{-1^2}{2!} + \frac{-1^3}{3!} + \frac{-1^4}{4!} + ... \\ &= -1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} + ...\end{aligned}$$

2. **Quiz 3**

   Write the following series using sigma ($\sum$) notation.

   (a)
$$1 - 2\,\theta^2 + 4\,\theta^4 - 8\,\theta^6 + \dots$$

   Solution:

$$\sum_{n=0}^{\infty} (-2)^n\,\theta^{2n}$$

(b)

$$\frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \dots$$

Solution:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2}$$

3. **Quiz 3**

   Memorized Power Series
   Look up and memorize the power series for e^z, $\ln(1+z)$, $\sin z$, $\cos z$, and $(1+z)^p$. For what values of z do these series converge? Do these functions have singularities in the complex z plane? How are any singularities related to the region of convergence?

4. **Evaluate Gamma and Beta Functions**
   Find the value of the following functions at the indicated points. Use technology or tables, but make sure you know a way to do this.

   (a) $\Gamma(4.2)$

   (b) $\Gamma(-2.2i)$

   (c) $B(4.2, -2.2i)$

REQUIRED:

5. **Integrals Involving Gamma and Beta Functions**
   Evaluate the following integrals using gamma and beta functions. It is **not** sufficient to look up the answer in a table of integrals or to use *Mathematica* or *Maple*, but you can use these resources to get a hint, if necessary.

   (a) $\int_0^\infty x^2 e^{-x^2}\, dx$

   (b) $\int_0^{\frac{\pi}{2}} \sqrt{\sin^3 x \cos x}\, dx$

   (c) $\int_0^1 x^2 (1-x^2)^{\frac{3}{2}}\, dx$

   (d) $\int_0^1 (\ln x)^{\frac{1}{3}}\, dx$

6. **Power and Laurent Series**
   Consider the function

$$w(z) = \frac{1}{z^2 - 5z + 6}$$

   (a) Find a power or Laurent series for the function $w(z)$ above, expanded around $z = 0$, for each annular ring where a series is defined.

(b) Find a power or Laurent series for the function $w(z)$ above, expanded around $z = 2$, for each annular ring where a series is defined.