PH461 Math Methods Capstone Homework 4

Due 4/25/16 3:50 pm

PRACTICE:

1. **Quiz 4**

   Find the total differential of each of the following equations or expressions. (This is your opportunity to review and master all the derivative rules that you may have forgotten. Do this exercise by hand.)

   (a)
   $$pV = Nk_BT$$

   (b)
   $$y = A\sec\theta e^{im\phi}$$

   (c)
   $$S = Nk_B\left\{\ln\left[\frac{V}{N}\left(\frac{mU}{3\pi N\hbar^2}\right)^{\frac{3}{2}}\right] + \frac{5}{2}\right\}$$

   (d)
   $$\ln\left(e^{\frac{\mu B}{k_BT}} + e^{-\frac{\mu B}{k_BT}}\right)$$

REQUIRED:

2. **First-Order, Separable Differential Equations**

   Find the general solution of each of the following differential equations. Use Boas Chapter 8 Section 2 for review of an appropriate technique.

   (a)
   $$y' = x^2/y$$

   (b)
   $$xy' - xy = y$$

   (c)
   $$xy' - xy = x$$

3. **Quadratic Drag**

   An object is moving in one direction which is only subject to one force, quadratic drag, given by
   $$-cv^2$$
   where v is the velocity of the object and c is a constant. Write the equation of motion for this object and solve for the velocity at any time t for an initial velocity of v_0.

4. **First-Order Exact Equations**

For the following equations, determine whether or not the equation is exact. If it is not exact, find an integrating factor which will make it an exact equation. Solve each equation.

(a)
$$(2xy + x)dx + (x^2 + y)dy = 0$$

(b)
$$y^2(\ln x + 1)dx + 2xy \ln x dy = 0$$

(c)
$$(3x + 2)ydx + x(x + 1)dy = 0$$

5. **First-Order Linear Equations**

Solve the following first-order linear differential equations using integrating factors. Refer to Boas 8.3 or Schaum's Outlines scan on course website.

(a) $y' + y = x$

(b) $xy' + y = x \cos^2 x$

(c) $\frac{dx}{dy} = \cos y - x \tan y$ (Boas 8.3.12)

6. **Maxwell Relation**

The First Law of Thermodynamics can be written in inexact differentials as $dU = \delta Q + \delta W$ which can be written in terms of exact differentials as $dU = TdS - pdV$ where U is internal energy of the system, T is temperature, S is entropy, p is pressure, and V is volume. For this expression to be exact, what relationship between T, S, p, and V must be true?