PH461 Math Methods Capstone Homework 6

Due 5/9/16 3:50 pm

PRACTICE:

1. **Quiz 6**
   For a given change of variables, e.g. $z = e^x$ or $u = 2\sqrt{x}$, find

$$\frac{d}{dx} \qquad \text{and} \qquad \frac{d^2}{dx^2}$$

REQUIRED:

2. **Wave Equation**
   Consider the equation

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial t^2} = 0$$

   (a) Change variables in the equation above to new coordinates u, v defined by

$$u = \frac{1}{2}(x + t) \qquad\qquad v = \frac{1}{2}(x - t)$$

   .

   (b) From your new equation, show that the general solution can be written

$$\phi(u, v) = f(u) + g(v)$$

   where f and g are arbitrary functions.

   (c) In your solution, change variables back to x and t. Interpret your solutions in terms of left-moving and right-moving waves.

3. **Second Derivative Change of Variables**
   If $x = e^u \cos\theta$ and $y = e^u \sin\theta$, show that

$$\frac{\partial^2 \phi}{\partial u^2} + \frac{\partial^2 \phi}{\partial \theta^2} = (x^2 + y^2)\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right)$$

4. **Polar Form of Cauchy-Riemann Equations**
   From the rectangular form of the Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \text{and} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

   derive the polar form:

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad \text{and} \qquad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

   Hint: This problem is essentially an exercise in the partial derivative chain rule.

5. **RL Circuit**

   In an RL circuit a resistor and inductor are connected in series. Using Kirchoff's Law, a differential equation describing this circuit can be written as $L\frac{dI(t)}{dt} + RI(t) = V(t)$. Assuming the voltage is not dependent on time, solve for the current as a function of time.

6. **Second Order Homogeneous Differential Equations with Constant Coefficients**

   For the following second order equations with constant coefficients, solve for the general solution. For reference, see Boas 8.5.

   (a)
   $$y'' + 2y' - 3y = 0$$

   (b)
   $$y'' + 9y = 0$$

   (c)
   $$y'' + 8y' + 16y = 0$$

7. **Second Order Inhomogeneous Differential Equations with Constant Coefficients**

   For the following inhomogeneous linear equations with constant coefficients, find the general solution for $y(x)$. See Boas 8.5-8.6 for reference.

   (a)
   $$y'' + 2y' - y = \sin x + \cos 2x$$

   (b)
   $$y''' - y'' - y' + y = 2e^x$$