**PH461 Math Methods Capstone Homework 7**

*Due Tuesday 5/16/16, 3:50 pm*

## PRACTICE:

1. **Quiz 7**

   **1-D Change of Variables**

   Consider a differential equation such as:

$$x^2\left(\frac{d^2y}{dx^2}\right) + 2x\left(\frac{dy}{dx}\right) - 5y = 0$$

You want to make the change of variable $x = e^z$ to find a differential equation with $z$ as the independent variable. As part of this process, you need to transform the derivatives

$$\frac{d}{dx} \qquad \text{and} \qquad \frac{d^2}{dx^2}$$

to derivatives with respect to $z$. This quiz asks you to do just part of the change of variables procedure, i.e. to transform these derivatives (for any given change of variables).

**Solution:**

$x = e^z$ means that $z = \ln(x)$. Therefore, $\frac{dz}{dx} = \frac{1}{x} = e^{-z}$.

$$\begin{aligned}\frac{d}{dx} &= \frac{dz}{dx}\frac{d}{dz} \\ &= e^{-z}\frac{d}{dz}\end{aligned}$$

and

$$\begin{aligned}\frac{d^2}{dx^2} &= \frac{dz}{dx}\frac{d}{dz}\left(\frac{dz}{dx}\frac{d}{dz}\right) \\ &= e^{-z}\frac{d}{dz}\left(e^{-z}\frac{d}{dz}\right) \\ &= e^{-2z}\frac{d^2}{dz^2} - e^{-2z}\frac{d}{dz}\end{aligned}$$

**REQUIRED:**

2. **Hermite Polynomials**
   The differential equation for Hermite polynomials $H_n(x)$ is given by

$$H_n'' - 2xH_n' + 2n\,H_n = 0$$

   Use series methods to find a polynomial solution of this differential equation for the case $n = 4$. For what values of $x$ is your solution valid?

3. **Laguerre Polynomials**
   The differential equation for Laguerre polynomials $L_m(z)$ is given by

$$zL'' + (1 - z)L' + nL = 0$$

   Find a polynomial solution of this differential equation for the case $n = 4$. For what values of $z$ is your solution valid?

4. **Hermite Polynomials**
   (a) Use *Mathematica* or *Maple* to find the first 5 Hermite polynomials.
   (b) Use Rodrigues' formula to calculate the first 5 Hermite polynomials. (You are encouraged to use *Mathematica* or *Maple* to help with the derivatives.
   (c) Look up two recurrence relations for Hermite polynomials and use them to find $H_3'(x)$ assuming that all you know is that $H_0(x) = 1$ and $H_1(x) = 2x$. Do this part of the problem by hand.