PH461 Math Methods Capstone Homework 9

Due Tuesday 5/31/16, 4:00 pm

PRACTICE:

1. **Quiz 9**

   Eigenfunction Expansions

   Consider the orthonormality relation for Hermite polynomials:

$$\int_{-\infty}^{\infty} H_m(x)H_n(x)e^{-x^2}\,dx = 2^n n!\sqrt{\pi}\delta_{m,n}$$

   The expansion of any smooth function $f(x)$ in terms of Hermite polynomials is:

$$f(x) = \sum_{0}^{\infty} c_n H_n(x)$$

   Write an integral expression for the coefficients c_n in this expansion.

   Similarly, you should be able to write down the condition for the coefficients in an eigenfunction expansion for **any** Sturm-Liouville problem.

REQUIRED:

2. **Quantum Particle in a 2-D Box**

   You know that the spatial eigenfunctions for a particle in a 1-D box of length $L = \pi$ are $\sin nx$. If you want the eigenfunctions for a particle in a 2-D box, then you just multiply together the eigenfunctions for a 1-D box in each direction.

   (a) Find the eigenfunctions for a particle in a 2-D box with sides of length L_1 in the x-direction and length L_2 in the y-direction.

   (b) Any sufficiently smooth spatial wave function inside a 2-D box can be expanded in a **double** sum of the product wave functions, i.e.

$$\psi(x,y) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} c_{nm}\ \text{eigenfunction}_n(x)\ \text{eigenfunction}_m(y)$$

   Using your expressions from part (a) above, write out all the terms in this sum out to $n = 3$, $m = 3$.

   (c) Find a formula for the c_{nm}s in part (b).

3. **Electrostatic Potential in a 2-D Box**

   Find and exact expression for (the two-dimensional toy model for) the electrostatic potential inside a 2-dimensional empty box with sides of length L_x, L_y if 3 of the sides are held at zero potential and one is at 5 Volts. To make is easy for the grader, please put one of the corners of the box at the origin, let the whole box live in the first quadrant (i.e. all positive values of the spatial coordinates) and make the side at 5 Volts be at $y = L_y$.

Make a plot of a reasonable approximation to the potential. State how you chose your approximation scheme and why.

4. **QM Particle in a 2-D Box**
Find an exact expression for the wave function for a quantum mechanical particle inside a 2-dimensional infinite potential well with sides of length L_x, L_y. The initial state is:

$$\psi(x, y, 0) = \frac{30}{\sqrt{L_x^5 L_y^5}}(L_x x - x^2)(L_y y - y^2)$$

I have chosen coordinates so that one of the corners of the box is at the origin and all of the box is in the first quadrant (i.e. all positive values of the spatial coordinates).

Plot an approximation for the probability density at $t = 0$ and at an interesting later time. Explain why you chose the later time that you did. Explain how you chose your approximation scheme and why.

5. **Waves on a Rectangular Drum Head**
Find the motion for a rectangular drum head with sides of length L_x, L_y if the drumhead is intially stretched in the shape:

$$\psi(x, y, 0) = 7(L_x x - x^2)(L_y y - y^2)$$

Assume that the initial velocity is zero:

$$\dot{\psi}(x, y, 0) = 0$$

Plot an approximation for the shape of the drumhead at $t = 0$ and at an interesting later time. Explain why you chose the later time that you did. Explain how you chose your approximation scheme and why.