Power from the Wind

Basic Concepts

The power in the wind, which is kinetic energy passing by per unit time, is

$$P = \frac{1}{2}\rho A v^3 \,,$$

where $\rho = 1.3\text{kg/m}^3$ is the density of air, A is the cross-sectional area in the plane perpendicular to the direction of motion and v is the speed of the wind. As an example, for $v = 10\text{m/s}$ the power per unit area is 700 W/m^2. From an analysis based upon laminar flow, incompressibility of air and conservation of energy, the maximum efficiency for extraction of power from the wind is 0.529 for an idealized wind turbine. The table lists the wind power applied to turbines of various radii for various wind speeds.

radius (m)	area (m^2)	wind speed (m/s)	Power (W)
1	3.14	1	2.04
1	3.14	5	255
1	3.14	10	2041
1	3.14	20	16,328
1	3.14	30	55,107
10	314	1	204
10	314	5	25,500
10	314	10	204,100
10	314	20	1,632,800
10	314	30	5,510,700
30	2826	1	1,836
30	2826	5	229,500
30	2826	10	1,836,900
30	2826	20	14,695,200
30	2826	30	49,596,300

The most efficient types of wind turbines are the two- and three-bladed horizontal axis wind turbine (HAWT) designs and Darrieus vertical axis wind turbine (VAWT) design. The two-blade turbine has the highest efficiency of about 40%, but its propensity to vibrate has resulted in the 3-blade design being more popular. HAWTs require generators situated behind the rotor s atop the towers whereas VAWTs have the advantage of allowing the generators to be on the ground. The disadvantage of a VAWT is its inability to begin moving by itself when the wind begins to blow. Multi-bladed designs used to drive water pumps are inefficient but operate in very light winds.

Modern HAWTs utilize yaw control to keep the turbine pointed into the wind, pitch control of the blades to enable constant power production when the wind speed exceeds some minimum specification and a mechanism to furl the blades when the wind becomes to strong.

Environmental effects are primarily noise and bird (mostly raptor) mortality. Neither has aroused significant concern.

Using Wind to Provide All U.S. Electrical Power

If we ignore the variability of the wind energy resource, we can estimate the land area needed to generate the average instantaneous U.S. electrical power usage. The country used 11.4Q = 11.4×10^{15} btu of electrical energy in 2000. [1] Since there are 8,760 hours in a year, this corresponds to 1.3×10^{12} btu/hr. Then, using 1 kW = 3.4×10^3 btu/hr, the average instantaneous usage is 0.38×10^9 kW = 380 GW.

The typical wind turbine will be assumed to be the Enron 1.5 MW model, which achieves its rated output when the wind speed is greater than or equal to 13 m/s. This is a giant 3-blade device with a diameter of 65 m, radius of 32.5 m, effective area of 3,318 m^2, and a rotor speed between 10 and 20 rpm. The power in a 13 m/s wind applied to this turbine is

$$P = \frac{1}{2} 1.3 \ \mathrm{kg/m^2} \times 3.318 \times 10^3 \ \mathrm{m^3} \ \times \left(\frac{13 \ \mathrm{m}}{s} \right)^3 = 4.74 \ \mathrm{MW} \,.$$

Since the rated power is only 1.5 MW, the efficiency is

$$\eta = \frac{1.5}{4.74} = 0.317 \,.$$

As shown in Figure 1, the turbines must be spaced sufficiently such that they are not affected by the wakes of the neighboring turbines. For these large 1.5MW units, the area required is approximately twice the rotor diameter multiplied by ten times the rotor diameter, or $A \approx 130$ m $\times$ 650 m $= 8.54 \times 10^4 \ \mathrm{m}^2 = 0.08 \ (\mathrm{km})^2$. So, such a wind farm could generate

$$\text{Power per unit land area } = \frac{1.5\mathrm{MW}}{0.08(\mathrm{km})^2} = 18.75\mathrm{MW}/(\mathrm{km})^2 \,.$$

Now, assuming that there exists a large flat area where the wind speed is always 13 m/s, all the electrical power used in the U.S. could be generated by an area of

$$\text{Area } = \frac{380\mathrm{GW}}{18.75\mathrm{MW}/(\mathrm{km})^2} = 2 \times 10^4 (\mathrm{km})^2 = (140\mathrm{km})^2 \approx (100\mathrm{mi})^2 \,.$$

This classic *back of the envelope* calculation is useful only as an indication that the wind resource and technology is readily available. For a real, large-scale system the variability of wind in any location and the variation in average wind speeds with location must be taken into account. Real wind farms will consist of turbines along ridges, in passes through mountains and hills, along shorelines and in some large flat regions. With coherent regional system design, wind resources generally available at different times of the day can be used to create a more constant flow of electrical power. Extensive use of photo-voltaic arrays would also help fill the gaps in wind resources during the day. Energy storage or on-demand fuel-fired power plants would also be required.

[1] US Annual Energy Outlook 2000, Energy Information Agency, Dept. of Energy.

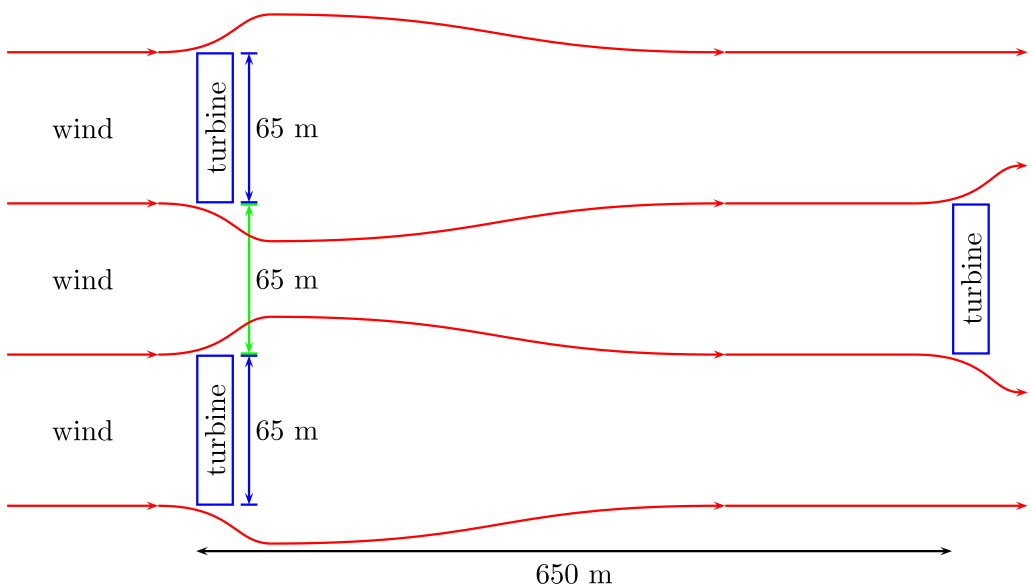

Figure 1: Wind farm using 65 m diameter turbines. The turbines are spaced to avoid interactions among them, so each turbine requires 130 m × 650 m of land.