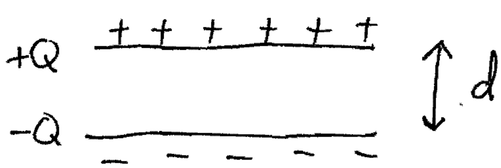
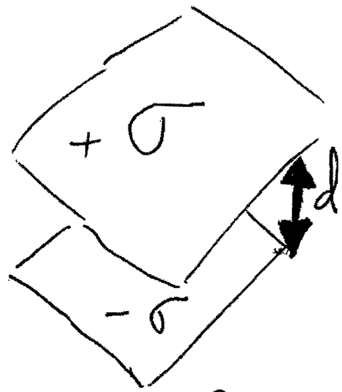
Capacitance:

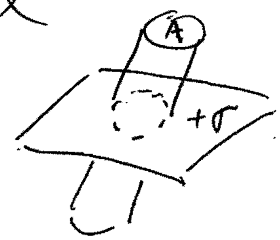
To derive C from first principles first use Gauss' Law to find $\vec{E}$, then find V, then find C.





$$\sigma = \frac{Q}{A}$$

(a) Find the electric field



$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\rightarrow \quad |E|A = \frac{\sigma A}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}}$$

Note that $\vec{E}$ is independent of distance.

(b) $$V = \int_0^d \vec{E} \cdot d\vec{l} \Rightarrow \boxed{V = \frac{\sigma d}{\varepsilon_0}}$$

(c) Recall that $Q = CV$, therefore

$$C = \frac{Q}{V} = \frac{Q\varepsilon_0}{\sigma d} = \frac{\cancel{Q} A \varepsilon_0}{\cancel{Q} d}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Questions:

(i) How do you physically change capacitance?

(ii) Can you derive the capacitance for cylindrical and spherical capacitors

(iii) This is a static problem, how do capacitors respond to driven frequencies? (More to come...)

Energy Stored in a Capacitor:

Recall that eV is a useful Energy unit. For a capacitor,

$$dU = q\,dV + V\,dq$$

Use $V =$ constant to arrive at

$$dU = V\,dq, \quad \text{so} \quad U = \int V\,dq$$

or

$$U = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C}$$

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C}} \quad \text{or} \quad \boxed{U = \frac{1}{2}CV^2}$$

Does this energy equation remind you of anything? Think spring energy...