Diode and Transistor Impedance

Definition of Impedance

The impedance $Z(\omega)$ is defined only in the frequency domain. For the simple case of a single frequency signal, with $V(t) = V(\omega)e^{i\omega t}$ and $I(t) = I(\omega)e^{i\omega t}$, the impedance of an object is defined as

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \ .$$

Capacitor Impedance

The current $I(t) = I(\omega)e^{i\omega t}$ is related to the charge $Q(t) = Q(\omega)e^{i\omega t}$ through

$$I(t) = \frac{d}{dt}Q(t) = i\omega Q(t) \ ,$$

where $I(\omega)$ and $Q(\omega)$ are complex. Since $V(t) = V(\omega)e^{i\omega t} = Q(t)/C$, the impedance is

$$Z(\omega) = \frac{Q(\omega)/C}{i\omega Q(\omega)} = \frac{1}{i\omega C} \ .$$

Inductor Impedance

The potential $V(t) = V(\omega)e^{i\omega t}$ is related to the current $I(t) = I(\omega)e^{i\omega t}$ through

$$V(t) = L\frac{d}{dt}I(t) = i\omega LI(t) \ ,$$

where $V(\omega)$ and $I(\omega)$ are complex. The impedance is

$$Z(\omega) = i\omega L \ .$$

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The current through the diode is $I(t) = I_\circ(e^{\alpha V(t)} - 1)$, with with $\alpha = e/\mathrm{k}T$ and $V(t)$ and $I(t)$ being positive when in the directions shown in the figure.

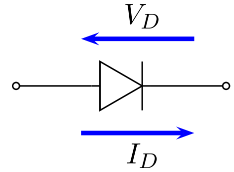

Figure 1: The diode and definitions of the current through and potential across.

It is possible to define a small signal impedance $Z(\omega)$ for a diode with no capacitance when $V(t) = V_\circ + v(\omega)e^{i\omega t}$ and $v(\omega) << V_\circ$. Begin by noting that

$$I(t) \approxeq I_\circ(e^{\alpha V_\circ} - 1) + \frac{d}{dV} I(V)\Big|_{V=V_\circ} v(\omega)e^{i\omega t} \ .$$

Then, since

$$\frac{d}{dV} I(V)\Big|_{V=V_\circ} = I_\circ \alpha e^{\alpha V_\circ} \ ,$$

The impedance is just

$$Z(\omega) = \frac{e^{-\alpha V_\circ}}{I_\circ \alpha} \ .$$

This is dimensionally correct as the dimensionality of α is potential^{-1} and the unit is Volt^{-1}. Notice that Z is a real resistive impedance and not a function of ω as no dynamic behavior of the diode has been considered. If the capacitance is known for $V = V_\circ$, then the total impedance is the parallel combination of the impedance above and the capacitive impedance.