PARADIGM 1: HARMONIC OSCILLATIONS

WORKBOOK FOR SECOND LABORATORY

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Damped Electrical Oscillations

by

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ABSTRACT

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FORMULATION OF HYPOTHESIS

Theory predicts that the response of an LRC circuit to an input harmonic function,

$$V_{input}(t) = V_0 \cos \omega t$$

is an output current that is also a harmonic function of time, with the same frequency as the input. The amplitude and phase of the current for a particular frequency can be predicted by calculating the impedance from circuit theory.

To test these predictions accurately we need to measure the response over a range of frequencies. The measurement frequencies should be chosen both above and below the system's resonance frequency.

FIRST OBSERVATIONS OF CIRCUIT RESPONSE

DIAGRAM OF APPARATUS *with detailed labels and close-ups*

CIRCUIT DIAGRAM

Circuit element	symbol	nominal rating *(units!)*

Using the nominal values of the circuit elements, we **predict** the following characteristics for the response of the circuit to an harmonic voltage input:

Predicted Quantity	Formula	Numerical Value *units!*
undamped frequency ω_o		
damping factor β		
damping time τ		
damped frequency ω_1		
Quality factor Q		

With the given values of the elements, the circuit will be ____________damped.

The relation between the output voltage V_{out} and the current I in the circuit is:

$$V_{out} =$$

Quantitative predictions for the circuit's response to a harmonic driving force are calculated by the section on "Harmonic Response of an LRC Circuit" in the *Maple worksheet on Fourier Integrals* (FourierIntegrals.mws).

PREDICTIONS FROM THE MAPLE WORKSHEET ARE INCLUDED AS PAGES ___________.

HARMONIC RESPONSE OF CIRCUIT

An harmonic voltage is sent into the circuit from the signal generator. The frequency is chosen to be close to the natural **resonant** frequency of the LRC circuit.

The oscilloscope displays the voltage as a function of time with a time scale unit of

t_{scale} = ________ . (*units!*)

The voltage scale is V_{scale} = _______.

The period of the input signal is T = _________ .

The amplitude of the input signal is V_0 = _________.

Sketch of INPUT signal *label axes quantitatively!*

Sketch of OUTPUT VOLTAGE

HARMONIC OBSERVATIONS

Based on the observed response as sketched on the previous page, we **observe** the following characteristics of the response of the circuit to an harmonic voltage input:

Observed Quantity	How to obtain it from the oscilloscope graph
period T	
lag time Δt	
input amplitude V_0	
output amplitude V_{out}	

COMPARISON OF THE PERIOD of the input and output voltages:

TEST OF LINEARITY HYPOTHESIS

The input amplitude V_0 is changed to be half its previous value.

quantity	predicted effect	observed effect
output amplitude V_{out}		
period T		
lag time Δt		

The relation between the output voltage and the current in the circuit is:

OBSERVATIONS FOR SEVERAL FREQUENCIES

case	period T	lag time Δt	input amplitude V_{o}	output amplitude V_{out}
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

ANALYSIS OF OBSERVATIONS

From these measurements we can find the **observed admittance *ZIV*** = 1/*Z*(ω) .

Derived quantity	Formula in terms of observed quantities
angular frequency ω	
Admittance magnitude |*ZIV*|	
Admittance phase shift δ	
Real admittance Re(*ZIV*)	
Imaginary admittance Im(*ZIV*)	

One additional quantity is needed, namely . Its value is = .

ANALYSIS FOR OBSERVED FREQUENCIES

case	angular frequency ω	Admittance magnitude |*ZIV*|	Admittance phase shift δ	Real admittance Re(*ZIV*)	Imaginary admittance Im(*ZIV*)
units					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

ASSESSMENT

Studying the data on the previous page(s), I conclude that the theory and the observations

NEW PREDICTIONS

The simple theory of impulse response for the LRC circuit predicts that an impulse voltage input to the circuit causes a current

We can test this prediction in the laboratory by measuring the current after an impulse voltage has been applied.

Using the nominal values of the circuit elements, we **predict** the following characteristics for the response of the circuit to an impulse voltage input:

Predicted Quantity	Formula	Numerical Value *units!*
undamped frequency ω_o		
damping time τ		
damped frequency ω_1		
initial current after impulse $I(t=0)$		
Quality factor Q		

IMPULSE RESPONSE OF CIRCUIT

A voltage pulse is sent into the circuit from the signal generator.

The oscilloscope displays the voltage as a function of time with a time scale unit of
t_{scale}= _________ . (*units!*)

The voltage scale is V_{scale} = ________.

The length of each input pulse is Δt = __________ .

The average voltage of each input pulse is V_o = __________.

Sketch of INPUT PULSE *label axes quantitatively!*

Sketch of OUTPUT VOLTAGE

Based on the observed response as sketched on the previous page, we **observe** the following characteristics for the response of the circuit to an impulse voltage input:

Observed Quantity	How to obtain it from the oscilloscope graph	Numerical Value *units!*
damping time τ		
damped frequency ω_1		
number of cycles in damping time		
initial current after impulse $I(t=0)$		

In our observations the circuit is ________________damped.

Compare the peak of the FFT spectrum with the measured frequency of the decaying oscillation. Estimate the Quality factor Q from the FFT spectrum. Record a trace of the decaying oscillation and save it to disk for later FFT analysis.

COMPARISON OF PREDICTED AND OBSERVED RESPONSE TO IMPULSE

Collecting the predictions from Page 9 with the observations from Page 11:

Quantity	Predicted Value	Observed Value
damping time τ		
damped frequency ω_1		
Quality factor Q		
initial current $I(t=0)$		

DISCUSSION OF DATA

Looking closer, what aspect of the data **agree**s with the predictions?

What aspect of the data **disagrees** with the predictions?

A simple way to understand this result is:

DISCUSSION OF IMPULSE AND HARMONIC DATA

Include:

comparison of impulse response versus prediction from circuit theory

comparison of observed harmonic response versus prediction from circuit theory (complex impedance)

comparison of observed harmonic response versus Fourier transform (FFT) of impulse response

Explain how your analysis supports each conclusion by referring to specific numbers in the tables and graphs above.

Do the results agree with your expectations? If not, how do they differ?

SQUARE WAVE RESPONSE

Show how the response of the LRC circuit to a square wave input is related to the impulse response. Do this in the Fourier transform space (i.e., frequency space) by using Fast Fourier Transforms (FFT). To do this, you will need to record 4 sets of data in the time domain: (i) impulse input, (ii) impulse response, (iii) square wave input, and (iv) square wave response. Choose a square wave frequency below the circuit resonance frequency. The data sets can be analyzed using FFT procedures in any package you wish to use. I recommend using Excel, since it can also be used to plot the results easily. A Maple worksheet (fft_lrc.mws) is also available on the course web site. In Excel, the FFT is available under Tools/Data Analysis; you have to load it if it is not there (just ask the Microsoft gnome for help on Fourier and follow the directions).

SUMMARY AND CONCLUSIONS

ACKNOWLEDGEMENTS

The measurements were taken in a group including

The computer analysis was performed by a group including

Helpful comments were provided in conversations with

BIBLIOGRAPHY

APPENDIX A

Output from the section on "Harmonic Response of an LRC Circuit" in the *Maple worksheet on Fourier Integrals* (FourierIntegrals.mws).

