PENDULUM PERIOD WORKSHEET

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The aim of this worksheet is to calculate **quantitatively** how the period T of a pendulum's oscillation depends on the amplitude of its motion and to explain the result **qualitatively**. This worksheet is to be included in your report for the pendulum experiment.

Consider a pendulum of mass M and moment of inertia I, whose center of mass is a distance L from the axle. Let the amplitude of the oscillation be θ_{max}.

Take the approach that we used in class for the mass on a spring example: that is, calculate the period by integrating the time differential (position over velocity) over one oscillation. The velocity is found from an energy consideration of the system. Simplify the integral as much as you can. For example, identify the period for small angle oscillations and call that quantity T_0. You can check your result by making the small angle approximation and seeing if your result matches that found in class for the mass on a spring.

You should be able to write a simplified expression for the period T as a product of T_0 and an integral that depends on θ_{max}. The definite integral is known as an elliptic integral. Find the solution to this integral 2 ways:

1) Solve exactly by looking up the elliptic integral or doing it with Maple.
2) Approximate the integral using the small angle approximation, but keep the next term past what would give you the answer we found in class for the mass on a spring. Your result should look like $T = T_0\left[1 + f(\theta_{max})\right]$, where $f(\theta_{max})$ is a function of the amplitude.

Tabulate and graph your results. Also include the results in your pendulum report and compare with the experimental data.

Discuss the qualitative explanation for why the period depends on the amplitude as it does (as we discussed in class).

Here is a sample table for your results.

θ_{max}	T/T_o Exact	T/T_o Approx

Here is a sample graph for your results (same as in pendulum lab).