PH 451: Capstone in Quantum Mechanics

Homework 6

Due 2/15/08

1. Consider particle #1 with spin 1/2 (s_1 = 1/2) and particle #2 with spin 1 (s_2 = 1).

   (i) List the possible values of total spin for this two-particle system, and for each, the projection of the spin on the z-axis. How many possible states are there for each total spin value?

   (ii) We will now find the eigenstates of the total spin and total spin projection on the z-axis. $\hat{S}^2 = \hat{S}_1^2 + \hat{S}_2^2 + \left(\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-} + 2\hat{S}_{1z}\hat{S}_{2z}\right)$ will be useful.

   (a) Prove that the state $\left|s_1 = \frac{1}{2}, m_{s1} = \frac{1}{2}; s_2 = 1, m_{s2} = 1\right\rangle$ is an eigenstate of the total spin and total spin projection on the z-axis, with eigenvalues $s(s+1)\hbar^2$ and $m_s\hbar$ respectively where $s = \frac{3}{2}$ and $m_s = \frac{3}{2}$.

   (b) Operate on this state with the lowering operator for the TOTAL spin to generate the next eigenstate of the total spin and total z-projection. Prove that the state you have generated is indeed an eigenstate and find s and m_s.

   (c) (Optional) Continue in this way until you have found all four states and their eigenvalues.

   You should get a table like this (partially filled in for you):

Total spin eigenstates	Combination of single particle spin eigenstates	s	m_s
$\left\|s = \frac{3}{2}, m_s = \frac{3}{2}\right\rangle$	$\left\|s_1 = \frac{1}{2}, m_{s1} = \frac{1}{2}; s_2 = 1, m_{s2} = 1\right\rangle$	3/2	3/2
$\left\|s = \frac{3}{2}, m_s = \frac{1}{2}\right\rangle$	$\sqrt{\frac{1}{3}}\left\|m_{s1} = -\frac{1}{2}; m_{s2} = 1\right\rangle + \sqrt{\frac{2}{3}}\left\|m_{s1} = \frac{1}{2}; m_{s2} = 0\right\rangle$	3/2	1/2
$\left\|s = ?, m_s = ?\right\rangle$	?	?	?
$\left\|s = ?, m_s = ?\right\rangle$	?	?	?

2. A particle is in a state that has orbital angular momentum quantum numbers $\ell = 1; m_\ell = 0$.

   What is the physical meaning of these numbers?

   Suppose you measure the orbital angular momentum component along the x-direction. What might you measure? And with what probability?

   Suppose such a measurement yields one unit of angular momentum along the x-direction. You subsequently measure the orbital angular momentum component along the z-direction.

   What might you measure? And with what probability?

3. (Goswami 17.A6) Consider two electrons of spin 1/2 and $\ell = 1$.

a. What are the possible values of the quantum number for the total orbital angular momentum $L = L_1 + L_2$?

b. What are the possible values of the quantum number for the total spin $S = S_1 + S_2$?

c. Now find the possible quantum numbers j for the total angular momentum $J = L + S$.

d. What are the possible values of the total angular momentum of each particle $j_1 = L_1 + S_1$ and $j_2 = L_2 + S_2$?

e. Find the possible values of j from the combination of j_1 and j_2 and compare with the result of part c.