PH 451: Capstone in Quantum Mechanics

Homework 8

Due 2/29/08

1. a) Consider the following states:

   $^1D,\ \ ^2P,\ \ ^4D$

   What J values are associated with each?

   b) Consider the states:

   $^1D,\ \ ^3P,\ \ ^3S$

   Given that each state consists of 2 equivalent (*i.e.*, in same nl state) electrons, are these states allowed by the Pauli Exclusion Principle?

2. Calculate the direct and exchange integrals (J, K in class, A, B in text) for the helium excited states (do n = 2 only). You may find the following useful:

$$\frac{1}{\left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right|} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{4\pi}{2\ell+1}\frac{r_<^{\ell}}{r_>^{\ell+1}}Y_{\ell m}^{*}(\theta_1,\phi_1)Y_{\ell m}(\theta_2,\phi_2),$$

   where $r_>$ stands for the larger of the two distances r_1 and r_2, and $r_<$ the smaller.

3. Review the calculation of the spin orbit and relativistic corrections for the hydrogen atom (Griffiths, Section 6.3) and be certain you understand the origin of Equations 6.57, 6.65, and 6.67. You don't have to turn anything in, but you must understand it.
   Now, armed with your complete understanding, calculate the size of the following (for hydrogen): (for a-d, tabulate your results and give answers in three forms: theoretical in terms of $\alpha^n mc^2$, in *eV* or *meV*, and in *GHz*)

a) The energy difference between the n = 1 and n = 2 states BEFORE any perturbations were considered.

b) The correction to the n = 1 and n = 2 states due to spin-orbit coupling. Note that the formula we derived in class is problematic for $\ell = 0$. Show that if you set $j = \ell + \frac{1}{2}$ and then use $j = \frac{1}{2}$, the problem goes away. (This is the Darwin term we talked about, but go ahead and call it spin-orbit here.)

c) The correction to the n = 1 and n = 2 states due to the relativistic term.

d) The total correction to these states, *i.e.,* the fine structure correction.
e) What wavelength resolution must your detector have to be able to able to resolve the two lines in the n = 2 to n =1 transition? Be careful here. When you include the correction, you will find that it is very small compared to the unperturbed value. Be sensible about how to include the effects.
f) Is it important to use the reduced mass of the electron in your calculations or is it OK to use the free mass?