# PH 451: Capstone in Quantum Mechanics

## Homework 9

## Due 3/7/08

1. Derive the expression for the energy of the hydrogen atom levels in a strong external magnetic field. Proceed as follows:

(i) If the applied magnetic field is "strong", what is it "strong" compared to? Give a numerical estimate.

(ii) If the external field is strong, we can ignore (for the moment) the spin-orbit (S-O) and relativistic effects. Consider the Zeeman Hamiltonian $\hat{H}_{Zeeman} = \frac{e}{2mc} B_{ext}(L_z + 2S_z)$ as a perturbation of the field-free Hamiltonian (excluding S-O and relativistic terms), and calculate the correction to the energy levels. (Why must you use the $|n, \ell, m_\ell, s, m_s\rangle$ basis?)

(iii) Now consider the spin-orbit and relativistic effects as perturbations to the field-free plus Zeeman Hamiltonians and show that these fine structure corrections result in:

$$E_{fs} = \frac{mc^2\alpha^4}{2n^3}\left\{\frac{3}{4n} - \left[\frac{\ell(\ell+1) - m_\ell m_s}{\ell(\ell+\frac{1}{2})(\ell+1)}\right]\right\} \text{ (at least for states where } \ell \neq 0\text{)}$$

(iv) Draw an energy level diagram for the 2*p* states of hydrogen, showing how the degeneracy is lifted in the presence of a strong external field.

2. Calculate the lifetime (in seconds) of each of the four $n = 2$ states of hydrogen ($|n, \ell, m\rangle$). The lifetime is the inverse of the spontaneous emission rate (Einstein A coefficient).

3. A particle of mass $m$ is initially in the ground state ($E_1$) of an infinite square well of width $a$. From $t = 0$ to $t = T$, the potential is perturbed so that it becomes

$$V(x) = \begin{cases} V_0 & \text{for } 0 \le x \le a/2 \\ 0 & \text{for } a/2 \le x \le a \\ \infty & \text{elsewhere} \end{cases}$$

where $V_0 << E_1$. Find the probability that the energy after time $T$ is measured to be $E_2$.