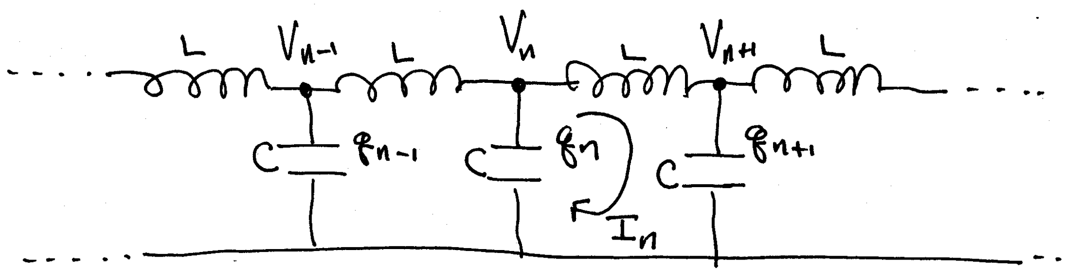
PH427/PH527 PERIODIC SYSTEMS

OSU Department of Physics
Spring 2008
Instructor: William Warren

Homework #2
Assigned: Friday 4/4/08
Due: Friday 4/11/08 1:00 pm

1. An electrical transmission line can be represented as a periodic series of equal inductances *L* and capacitances *C* arranged as shown below.



The potential at the *n*th node is V_n, q_n is the charge on the *n*th capacitor, and I_n is the current in the *n*th loop, defined as shown in the diagram. Using the equations

$$V_n - V_{n+1} = L\frac{dI_n}{dt} = \frac{q_n}{C} - \frac{q_{n+1}}{C} \qquad and \qquad \frac{dq_n}{dt} = I_{n-1} - I_n ,$$

show that the transmission line is essentially a low-pass filter with a maximum ("cut-off") frequency ω_{max} for propagating modes.

(a) Find an expression for ω_{max} in terms of *L* and *C*.

(b) Suppose *L* and *C* are the inductance and capacitance, respectively, per unit length of the line. What is the speed of propagation of waves with frequency far below the cut-off frequency?

2. For the system of coupled pendulums you studied in Homework #1, problem 3, describe, quantitatively and qualitatively, the motion that results if the system is driven at an angular frequency ω that is *lower* than $(g/L)^{1/2}$.

3. Consider a 1-dimensional diatomic lattice. The atomic spacing is 0.3 nm, the maximum frequency of the acoustic branch is 10^{13} rad/s, and the long-wavelength frequency of the optic branch is 1.594×10^{13} rad/s.

   (a) Calculate the speed of sound in this 1-dimensional crystal. First explain which region of the dispersion relation you will use to calculate this and why. Then find an expression for the velocity that makes use of the parameters given in the problem.

   (b) If the atoms are Rb and Br, what is an effective spring constant for the bond between them?

4. Total energy and specific heat (as a function of temperature) for a 1-D monatomic chain.

   (a) What is meant by a density of states function? Find the particular form for <u>the one-dimensional monatomic chain of *N* masses separated by a distance *a*</u> and sketch it as a function of the appropriate variable in (i) *n*-space, (ii) *k*-space and (iii) ω-space. Does a divergent density of states function present a problem?

   (b) What is meant by a distribution function (sometimes called an occupation function)? What particular function do we use for this system? There are two others that are important for other systems. What are they?

   (c) Evaluate the total energy as a function of temperature for the system under consideration, and hence discuss the behavior of the phonon specific heat as a function of temperature. It may be too hard for a closed-form solution. Consider numerical solutions in different temperature regimes. Your results are best expressed on a log-log plot to bring out the different exponents in the temperature dependence.

5. Calculate the total energy of a system of *N* particles in a one-dimensional infinite square well potential at zero temperature. These particles are subject to the following rules:

   • No more than two particles per state

   • Particles don't interact. This simply assures that the states in question are indeed those we found in PH424, unmodified by the presence of many particles.

